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The Executive Processes of Cognitive Strategies in Mathematically Learning Disabled and Average-Achieving Adolescents

By

(C) KAREN S.Y. LEE

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
AND RESEARCH IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF EDUCATION
IN
SPECIAL EDUCATION

DEPARTMENT OF EDUCATIONAL PSYCHOLOGY

EDMONTON, ALBERTA
SPRING, 1986

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled The Executive Processes of Cognitive Strategies in Mathematically Learning Disabled and Average-Achieving Adolescents submitted by KAREN S.Y. LEE in partial fulfilment of the requirements for the degree of Master of Education in Educational Psychology.

(Supervisor)

Fern Snart

Date March 19, 1986.

Abstract

Both Neo-behavioral and psychometric approaches play dominant roles in educational research, particularly in North America. Many product-oriented studies of learning disability (L.D.) focus on examining the weaknesses of L.D. individuals with only cursory glances at their strengths.

The process-oriented method, following the clinical case-study approach, became the focal point of this cognitive study on strategic thinking and decision making expressed by the executive processes of strategies found in the mathematically learning disabled (M.L.D.) and non-mathematically learning disabled 7th graders. The research method of this study generated some of Vygotsky's brilliant thoughts with challenging modifications based upon the technique of collecting verbal data and the innovative idea of designing the mathematics word problems with references from Ericsson and Simon (1980), Goodstein (1981), and Swanson's (1982) works. At the same time, Soviet studies on mentally handicapped children and their mathematical performance activated the self-generated design of two similar tasks. Each task involves four arithmetical operations (+,-,x,÷) found in three major categories of mathematics word problems, namely: ordinary problems that are familiar from classroom instruction, problems with surplus information, and problems with missing information.

Concurrent data from verbal protocols and written scripts were collected; whereas the specially designed checklist of cognitive strategies and task performance as well as some cognitively-based criteria for analyzing error patterns became the stepping-stone of accomplishing a systematic descriptive analysis for individual subjects.

The research findings indicated both qualitative differences and similarities in strategies monitoring among the subjects. Evidence was also presented that similar cognitive strategies were applicable across the task performance on four basic arithmetical operations and the three major categories of mathematics word problems designed for the study. It seems quite likely that cognitive strategies can serve as a means to facilitate the goal-directed task performance across situations or settings. No evidence of strategy deficiency was found in the M.L.D. subjects but the proficiency and consistency of executing task-appropriate strategies in various situations, particularly shown in problems with surplus or missing elements, challenged both types of subjects in this study. Moreover, the common assumption of strategy deficiency found among heterogeneous types of learning disabled individuals needs to be reexamined.

A more innovative and critical research trend in learning disabilities shifts to focus on the proficiency and consistency of executing strategic thinking and decisions flexibly across situations and tasks. Recommendations suggest recontextualizing school curricula by means of teaching the disabled youngsters a system of learning and thinking strategies. It requires an exploration of some cognitive-dialectical teaching techniques based upon the notion of mediated learning experience shared by Feuerstein, Meichenbaum, and Vygotsky. Educational intervention discourages a reproducing-oriented or a surface approach in learning. A meaning-oriented approach to teaching and learning is strongly advocated. It fosters purposeful learners and productive thinkers who know how and when to conduct and generate insights into their strategic decisions which are applicable across multiple academic tasks and real life situations.

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CHAPTER 1

INTRODUCTION

Cognitive processes and strategies control are complex phenomena. Their definitional diversities are problematic in many studies on cognition.

Kirby (1984) described processes as a subset of cognitive and mental functions. Kagan and Kogan (1970) have stated:

> the term "cognitive" has typically referred to mental activities in the sense of both product and process . . . . Cognitive process is a superordinate term, subsuming the more familiar titles of imagery, perception, free association, thought, mediation, proliferation of hypotheses, reasoning, reflection, and problem solving. All verbal behavior must be a product of cognitive processes. (p. 1275)

Lawson (1984) suggested that cognitive processes encompass two major components: knowledge and control of strategies, both of which are usually described as metacognition. They are indicated in the sense that a person is able to talk about specific strategies, to know how and when to execute strategies for actual performance, to control them in the course of planning cognitive actions, and to choose among alternative activities during task performance. The regulatory and executive functions of cognitive processes provide space for the working memory (Brown & Smiley, 1978; Gagné, 1983; Kirby, 1984; Newell & Simon, 1972).

Gagné (1974) explained that:

> a cognitive strategy is an internally organized skill that selects and guides the internal processes involved in defining and solving novel problems. In other words, it is a skill by which the learner manages his own thinking behavior . . . Undoubtedly, the efficacy of an individual's cognitive strategies exerts a crucial effect upon the quality of his own thought. (p. 29)

At the same time, strategies control refers to the process by which the learner monitors and changes strategies that become means to achieve a goal (Forrest-Pressley & Waller, 1984).

The quality of task performance, particularly the solving of mathematical problems, can be evaluated more accurately and objectively by aspects both of process and of product/outcome (Kantowski, 1977). There are many product-oriented studies concerned with the reading disabled but little has been done to examine how the executive processes of strategies influence the outcomes of task performance among the mathematically learning disabled at junior high school level.

Learning disabilities encompass heterogeneous groups of students with various learning problems. They are usually characterized as inactive learners who have problems in attention, organization, and monitoring cognitive strategies. Their disabilities are influenced mainly by their impulsive learning behavior, poor knowledge or insufficient practice in applying task-appropriate strategies flexibly and efficiently, and poor learning motivation due to chronic experiences of academic failure (Forrest-Pressley & Waller, 1984; Keogh & Margolis, 1976; Torgesen & Licht, 1983; Wong, 1982).

It has been suggested that the learning disabled do apply cognitive strategies, but that their strategies are inefficient, ineffective, or inflexible. It may be that strategies are operated with qualitative differences among them in a hierarchy of proficiency. The quality of task performance is also influenced by the background of conceptual knowledge; by the ways to recall and organize rules/schemata while devising a plan efficiently; and most importantly, by the executive processes of synthesizing and assimilating the rules (Gagné, 1983; Gerber, 1983; Lupart & Mulcahy, 1984; Mayer, 1975; Polya, 1957; Webb, 1979).

There are many studies of L.D. focusing on examining what the disabled learners cannot do or are deficient in, but very little is known about what they can do in comparison to average-achieving individuals, or how and why the execution and regulation of cognitive strategies influence the quality of their tasks' performances. Ecological validity has been one of the major issues central to research on learning disabilities as well as other areas of exceptionality (e.g., mental retardation).

Ecological validity

Within the realm of cognitive science, research efforts have been directed mainly toward examining problems of intelligence. IQ tests and achievement tests offer limited accuracy in predicting the school failure of the learning disabled. These types of tests have biased current attitudes, examinations, and treatments of youngsters with learning problems.

The problem regarding an apparent gap between experimental research and the real life contexts of the subjects has become a serious concern, particularly when heterogeneous subgroups of special populations are involved in studies with various orientations. Keogh (1976) maintained that more experimental studies were not necessarily the answer to the problems in learning disabilities. She commented that the experimentalists aimed to test effects of treatments as well as seeking homogeneous subjects to reduce sample variance:

> This approach works particularly well in agricultural studies and reasonably well in the animal laboratory; it is less achievable with human subjects, especially exceptional children. One solution, of course, is to move to "N of 1" studies. (p. 138)

Willems (1965) proposed the descriptive method as an ecological approach which provided nonexperimental ways to understand human behavior. Keogh (1976) supported Willem's (1965) comment that:

> It might be noted that astronomers, although they do not control or manipulate the subject matter of their studies, are recognized as scientists, even by operant conditioners. It seems likely that both descriptive and experimental approaches are necessary and compatible. Careful, systematic, reliable, and replicable descriptions frequently provide the basis for controlled manipulation in the experimental paradigm and, in fact, may be a necessary first step before experimental manipulation can occur meaningfully. (p. 138)

It seems very likely that some alternatives to experimental studies are needed, especially for research on learning disabilities.

Some alternatives for L.D. research

Most L.D. studies are overruled by the experimental paradigm in terms of emphasizing what should be examined and how to formulate experimental conditions to fit the model of study. This often results in superficial and inadequate experimental research. Some researchers stress correlation coefficients or single-variable studies which demonstrate repeatedly that specific behaviors can be modified, at least temporarily, under certain experimental conditions of reinforcement. Keogh (1976) commented metaphorically that one way to get drowned in conducting research was to put an overemphasis on statistically and methodologically sophisticated experimental designs which asked miniscule questions. At the same time, emphasis on labels, etiologies, and ex post facto designs has not provided remarkable insights into the nature of learning disabilities. It is quite true that:

> There is no magic in behavioral analysis, any more than there is magic in diagnostic assessment . . . . It is lack of clear conceptualization of the question which has led to inadequate, nonproductive, and "fuzzy" research in special education. (Keogh, 1976, p. 139)

Elkins (1976) explained the importance of conceptualizing learning disabilities:

> because, at the clinical level, the structuring of knowledge used in understanding a particular child reflects the observer's conceptualization of the antecedent-consequent relationship which is operating. (p. 152)

Elkin's (1976) suggestions coincide with Vygotsky's (1978) notion of utilizing phenotypic and genotypic approaches to study and interpret individuals' mental processes. Elkin (1976) suggested that phenotypic analysis is based upon external features and begins directly with an object's current features and manifestations. Genotypic analysis is related to its origin and causal dynamic basis rather than its outer appearance. He suggested that:

> two phenotypically identical or similar processes may be radically different from each other in their causal-dynamic aspects and vice versa; two processes that are very close in their causal-dynamic nature may be very different phenotypically . . . . We seek to understand the real links between the external stimuli and internal responses that underlie the higher form of behavior named by introspective descriptions . . . . By necessity, objective analysis includes a scientific explanation of both external manifestations and the proces under study. (p. 62-63)

Vygotsky (1978) remarked that researchers have focused on the reaction time of the "fossilized" responses they study, not on the learning processes or the content of the reaction itself. He advocated that the study of cognitive functioning did not require the experimenter to provide subjects with external or artificial means in order to accomplish the given task as expected. He commented that:

> the experiment is equally valid if, instead of giving children artificial means, the experimenter waits until they spontaneously apply some new auxiliary method or symbol that they then incorporate into their operations . . . . We might trace the development of arithmetic skills in young children by making them manipulate objects and apply methods either suggested to them or "invented" by them... We study not only the final effect of the operation, but its specific psychological structure. (p. 74)

In summary, there is voluminous laboratory research concerning learning disabilities. Laboratory research does not necessarily mean experimental research, but can also refer to research in which instruction/intervention is not involved, such as studies of process and strategies variables. Descriptive studies of learners' behaviors, particularly among the special populations, in natural classroom/school settings are crucial to ecological validity. This can be done by means of following a clinical paradigm while observing the subjects engaged in a specific task and asking them unobtrusive yet probing questions about their activity.

The present study utilized the descriptive case study approach to explore the executive processes of cognitive strategies found in mathematically learning disabled and average-achieving seventh graders during their task performance. Another focus of this study stressed how and why these two types of subjects were different and/or similar qualitatively in applying some specific kinds of strategies for solving the following three major categories of mathematical problems: ordinary problems (familiar from classroom instruction); problems with surplus information; and problems with missing information. It was believed that the analyses of the subjects' verbal protocols and written scripts would provide more current and detailed evidence as to how they organize plans for sequential/subsequent behaviors as well as examine their processes of executing and regulating strategies during task performance.

CHAPTER 2

SELECTIVE REVIEW OF THE LITERATURE

Metacognitive knowledge and strategy-monitoring

Various definitional approaches to metacognition magnify the conceptual conflict among the researchers with different foci of interests.

Flavell (1976) suggested that:

> metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them e.g., the learning-relevant properties of information or data ... Metacognition refers, among other things to active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective. (p. 232)

Brown and Palincsar (1982) defined metacognition within two major categories: 1. knowledge about cognition, and 2. the regulation of cognition. Knowledge about cognition:

> involves conscious access to one's own cognitive operations and reflection about those of others; it is a form of declarative knowledge about the domain 'thinking'. (p. 1)

They continued to explain that the regulation of cognition involves:

> planning activities prior to undertaking a problem (e.g. predicting outcomes, scheduling strategies, and using forms of vicarious trial and error), monitoring activities during learning (monitoring, testing, revising, and rescheduling one's strategies for learning), and checking outcomes (evaluating the outcome of any strategic action in terms of criteria of efficiency and effectiveness). (p. 12)

Metacognitive activities are then concerned with both conscious reflection and regulation of cognitive knowledge and strategies. Lawson (1984) advocated distinguishing between metacognitive knowledge and executive processes as two dimensions within cognitive processes.

He perceived metacognitive knowledge as domain- or task-specific and as characterized by conscious awareness. The marshalling and regulating functions define the controlling feature of executive processes.

Kirby (1984) commented that cognitive processes encompass two major categories: processes and strategies. He explains that processes refer to cognitive performance, such as, encoding and storing of information, and strategies refer to controlling the use of these processes. Strategies are techniques or rules which facilitate the acquisition, integration, storage, and retrieval of information in all situations and settings. They can be narrow and situation specific or as broad and generalized as a cognitive style (Alley & Deshler, 1979; Kirby, 1984).

Lawson's notion of executive processes includes both strategy control and actual task performance, whereas Kirby indicates the difficulty in making the distinction between strategy control and the executive process of task performance.

> Strategies have process aspects, and vice versa. Furthermore, when the cognitive system begins to perform a realistic task, the distinction is hard to maintain. (Kirby, 1984, p. 5)

Kirby (1984) advanced his discussion of automaticity which separates strategies into well-established and to-be-constructed ones. His idea agrees with Lawson's (1984) belief that not all executive processes are conscious, controlled processes. Many executive operations are automatically expressed and therefore are not conscious or reportable:

> The skilled processor of information may show little evidence of executive processing simply because he or she is an expert at a given task. The novice may, on the other hand, be overtly executive on the same task because the task is new and the novice's processing requires more careful planning, analysis, and modification than that of the expert. (p. 92)

Lawson (1984) also believed that the application of efficient strategies does not guarantee success because the outcome of task performance does depend on the individual's knowledge base and experiences.

It has been suggested that children with learning problems suffer either structural or production deficiency. Flavell (1970) explained that mediational/structural deficiency referred to a learner's inability to employ a potential mediator/strategy even when he/she was specifically instructed to do so; production deficiency meant failure to use a strategy, though this could be remediated by adequate training. A mediational deficiency is found when a learner is trained to employ the required strategy, but this does not mediate performance. Production deficiency is related to inadequate use of control processes but can be remedied by appropriate strategy training. Both kinds of deficiencies are described as structural and process deficits, respectively (Gerber, 1983).

There are general and specific factors which affect a learner's use of strategies. Specific factors refer to the efficiency and sophistication of using a given strategy, and general factors refer to the learner's ability or intention to plan performance in advance. The planning aspect found in the general factor is relevant to the effective performance of all mediational activities instead of relating to any one particular activity. The sensitivity of knowing when a situation calls for voluntary and intentional efforts as well as knowing the influence of specific acts on task performance depend on the variables of individual, task demand, and strategy (Flavell, 1970, 1978). Further research is required for exploring what and how the quality of task performance, particularly found among the individuals with learning problems, would be influenced by the executive processes of cognitive strategies.

Cognitive performance between L.D.s and non-L.D.s

Narrowly but clearly-defined target populations, especially on descriptions of subjects with specific learning problems, are important before proceeding to any scientific examination. Various definitions of learning disability have been put forth. Some theorists focus on educational factors, others stress perceptual-motor factors, while still others follow a clinical approach (Cruickshank, Deutsch, Morency, Strother & Wepman, 1975; Hagin, Beecher & Silver, 1982; Haring & Bateman, 1977). Whatever the relationship between learning disabilities and psychophysiological aspects or information processing deficiencies, academic deficit and presumed average intelligence are common components in all of these definitions.

There are numerous definitions of learning disabilities. This section will attempt to summarize these and will highlight some of the most commonly accepted criteria. Learning disability is manifested by significant difficulties in the acquisition and use of thinking, listening, speaking, reading, writing, or mathematical abilities. Students with learning problems manifest an educationally significant discrepancy between their estimated intellectual potential and actual level of performance related to deficits in the learning processes

(Hammill, 1981). The National Advisory Committee on Handicapped Children (1968) proposed that learning disability may not be attributed to a visual, hearing or motor handicap, mental retardation, emotional disturbance, environmental, cultural or economic disadvantage. Common characteristics of the learning disabled population, such as reading problems, language difficulties, behavioral p roblems, process disorders, and visual motor difficulties, can be categorized as different areas of disorder: motor activity, emotion, perception, symbolization, attention, and memory.

Algozzine and Ysseldyke (1983) in their critiques relating to definitional diversity of learning disabilities suggested that most definitions of learning disabilities describe a population with a severe discrepancy between achievement and intellectual ability in various areas. The extent or severity of the low achievement, however, is often not clearly specified. They suggested that:

> the current reliance on unspecified degrees of discrepancy between ability and achievement is deceiving and may be ill-founded as a basis for a separate category of children to receive special education services. In fact, there may be an equally large number of children exhibiting similar degrees of school achievement not commensurate with their measured ability who are not categorized and therefore are not receiving special education services even though they are eligible for them under the current conceptual scheme represented by the category of learning disabilities. (p. 246)

They argued that various assessing instruments attempt to make learning disabilities more sophisticated and that:

> ... this oversophistication has spilled into the decision-making process is obvious when practitioners must rely on beta weights, estimated true scores, asymmetrical confidence intervals, and/or mile long regression equations to determine who is learning disabled. (p. 246)

These statistical scores only predict educational placement with slightly better-than-chance accuracy. They advocated the need for a new perspective on assessment and intervention. Both researchers also (1983) suggested that assessment must begin with implementing instructional/behavioral interventions as well as change from referral-to-placement to referral-to-intervention. There are heterogeneous subgroups of learning disabilities in which mathematical learning disability will specifically be emphasized in the following presentation.

In line with the specific population under examination within the present study, this section will shift to a discussion of mathematical learning disability. Mathematically learning disabled students are described as lacking specific ability to perform written or mental calculations despite average or better intelligence, adequate motivation and appropriate educational opportunities. Henderson and Pingry (1953) suggested three major processes through which an individual defines a mathematics problem:

1. The individual has a clearly defined goal of which he/she is consciously aware.

2. Fixed patterns of behavior or habitual responses are not sufficient for removing hindrance toward the goal.

3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible hypotheses/solutions, and tests them for feasibility.

Polya (1957) developed four phases for mathematical problem solving:

1. understanding the problem
2. devising a plan
3. carrying out the plan, and
4. looking back

His model provides some guidelines for organizing instruction but it does not specify the cognitive processes involved in mathematical problem solving. Each phase of mathematical problem solving requires deliberate and systematic mental activities. Soviet researchers have described mathematical problem solving as requiring an application of the analytic-synthetic method (Kalmykova, 1975; Bogolyulov, 1972; Mikhal'skii, 1975; Shchedrovitskii, 1972). This is in agreement with Polya's (1957) idea that analysis requires invention while devising a plan; whereas synthesis allows execution of the plan.

Lester (1980) introduced 4 major factors which influence the success of mathematical problem solving:

1. the problems,
2. the problem solver,
3. the problem solving process, and
4. the problem solving environment.

They are similar to Flavell's (1979) notion of task demands (the problem), person variable (the problem solver), and cognitive strategies (the problem solving process). Lester (1980) however did not explain how the four factors interact during the course of problem solving. A more in-depth discussion of the interactions among person, task, and strategy will be presented in the last section of this chapter.

Students with learning problems in mathematics are sometimes labelled as dyscalculic, and dyscalculia itself does not seem to be well-defined (Austin, 1980; Ceci & Peters, 1980; Cohn, 1968; Levy, 1979). Kocs (1974) defined dyscalculia as:

> a structural disorder of those parts of the brain that are the direct anatomico-physiological substrate or the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions. (p. 165)

Johnson (1979) argued that neuro-physiological problems should be excluded in defining mathematical learning disability. These problems must be dealt with separately prior to any assault on a specific type of mathematical learning disability because:

> if an individual is incapable of seeing, hearing, feeling, handling, moving or any other purely physical function necessary for being involved in arithmetic learning, ... the problem is one of inability to receive or become involved in necessary learning experiences, and therefore is a matter of lack of physiological integrity which must be resolved. (p. 33)

He further suggested that certain categories, such as retarded, economically deprived, and the emotionally disturbed, should be excluded due to the fact that:

> they involve either some inability of the child to ever profit from normal classroom arithmetic performance, such as the retarded individual who needs special goals as well as methods; or they represent those who simply have never had the opportunity to experience learning in a typical situation. Both represent peculiar and significant problems, but neither are learning disabilities. (p. 34)

This study defines the mathematically learning disabled as learners whose mathematics performance is two years or more behind in respect to chronological age and grade equivalent despite having at least fourth grade or above reading level, an average I.Q. of 95 or above, adequate motivation and appropriate educational opportunities with no emotional, neurophysiological, psychological, and mental retardation problems.

There are several prevalent problems found in the mathematically learning disabled as outlined by various writers, including performing arithmetic operations, deciding the process for problem solving, visual or auditory figure-ground relationships, expressive and/receptive language, sequencing, short- and/or long-term memory. On the other hand, many of them may show extraordinary auditory abilities and may even excel in reading and score higher on verbal than on non-verbal parts of standardized tests (Bartell, 1975; Bley & Thornton, 1981; DeRuiter, 1982; Johnson & Myklebust, 1967; McEntire, 1981; Reisman & Kauffman, 1980). This lends support to Jansky's (1965) argument that:

> one cannot isolate one single factor as being the sole determinant in arithmetic disabilities. (p. 257)

The origins of mathematical deficit are still uncertain. Hammill and Bartell (1974) believed that students with mathematical deficit experience difficulties in abstract or symbolic thinking, in terms of the ability to conceptualize the relationship between numerals and objects, the relationships between units of measurement, and the structure of the number system. They also suggested that ineffective instruction may be at times shown in students who have better performance in arithmetic concepts than in the skills acquired from specific instruction, such as understanding the concepts of doing four basic mathematical operations but making consistent errors in "carrying" while doing division. Some major influential factors of mathematical deficit are related to deficiencies in reading or the use of mathematical language (verbal or graphic), performance errors, in relating to the products or final answers, and anxiety, as well as cognitive developmental differences in associating with the process of knowledge acquisition and application (Cohn, 1961; Critchley, 1970; McEntire, 1981).

Developmental studies have demonstrated age differences in aspects of cognition. Age-related performance differences have been thoroughly studied on memory tasks. Younger children are usually found to be inferior in memory performance than the older individuals. This has been discussed as due to the fact that younger children have less knowledge and experience of strategies available and applicable for meeting various task demands. They usually learn and recall things incidentally but seldom carry out retrieval activities deliberately, such as verbal rehearsal. The semantic or conceptual systems of older individuals are more advanced developmentally in content and structure which allows them to store, retain, and retrieve inputs better than younger ones (Flavell & Wellman, 1977). An earlier study conducted by Ornstein, Naus, and Liberty (1975) indicated that younger children show a stronger recency effect in free recall of 18 words than older children do. The executive process of rehearsal becomes an important factor which influences their performance, producing such age-related differences.

Flavell and Wellman (1977) suggested that children will search the "internal and external world" in response to another's retrieval direction at a very young age.

A growing child will become more skillful in retrieving, planning, and integrating knowledge and strategies as well as being more attuned to internal "mnemonic sensations". This is evidenced by their growing ability to monitor and interpret concrete experiences. Brown (1978) indicated that cognitive strategies are characterized as efficient thinking which involves predicting, checking, monitoring, reality testing and coordination, and control of deliberate attempts to solve problems. Meichenbaum (1977), in his explanation of cognitive structure, referred to it as the organizing aspect of thinking that controls strategic thoughts and behaviors. He also suggested that it is the executive processor which determines when to interrupt, change, or continue thought.

Vygotsky's (1978) thesis of cognitive development stressed two dimensions of intellectual functioning: the actual and potential developmental levels. His concept of the zone of proximal development analyzes the relationship between learning and development. Actual developmental level indicates a child's mental functioning level that has been established as a result of certain already completed developmental cycles. Zone of proximal development is:

> the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by independent problem solving under adult guidance or in collaboration with more capable peers. (Vygotsky, 1978, p. 85)

Vygotsky (1978) suggested that:

> memory rather than abstract thought is the definitive characteristic of the early stages of cognitive development. However, in the course of development a transformation occurs, especially in adolescence... For the young child, to think means to recall; but for the adolescent, to recall means to think. Her memory is so "logicalized" that remembering is reduced to establishing and finding logical relations; recognizing consists in discovering that element which the task indicates has to be found. (p. 51)

Developmental differences are not the sole determinant of goal-directed task achievement. This is illustrated in Chi's (1978) finding that 10-year-old experts in chess performed better than adult novices. Cumulative learning through real-life experiences activate an individual's intellectual development (Gold, 1978; Steffe, Richards & Von Glasserfeld, 1979). Learning experiences are those which provide for the learner opportunities spontaneous growth through reducing the tendency to use a currently inadequate executive strategy, and those which are the basis for instruction, where the learner's thoughts are directed by someone who already knows a higher level strategy (Case, 1978).

Soviet studies provide some insights into how one might analyze special students' mental processes while solving some mathematical word problems. The special learners in these studies were not well-defined, but were usually described as mentally retarded students with brain damage and were found in the "auxilliary schools".

Mikhal'skii (1975) analyzed some common difficulties encountered by these students, such as, difficulties in choosing needed information and extracting the essential question in the problem from the given conditions. It is interesting to find out that these students followed a habitual pattern while solving the mathematics problems; for example, selecting numerical data, correctly or incorrectly, from pieces of the conditions given; or from individual words according to a formal feature, but not according to content; proceeding to solution by means of following the individual parts of the conditions, almost unrelated to the questions; hurrying to arrive at solutions according to a rote arithmetical operation without sufficient understanding of the meaning of the question and/or the conditions of the problem presented. Selecting a brief descriptive analysis from Soviet studies will provide some basic understanding of the seventh graders' mathematical performance:

> Katya B., having read problem 1 once, began to whisper something, continually moving her pencil along the lines of text, pretending she was thinking studiously, but for a long time did not move. Finally, she selected numbers for the first part of the conditions: "There are 10m of silk and 5m of satin. 5m more expensive than satin ...." From this data she incorrectly determined the numerical data for the second half of the conditions ("A meter of silk is more expensive than a meter of satin"): "10 minus 5 is 5. 5m more expensive. Having made up only two numbers, both unnecessary for solving the problem, the pupil began solving the problem". (Mikhal'skii, 1975, p. 48)

Word problems with surplus elements are usually longer in length and more complex in sentence structure. Studies show that syntactic complexity, including both sentence length and structure, has no negative effect of task performance (Goodstein, 1971; Muth, 1984). Most common errors occur when students use their "number crunching routines" to produce an answer, sensible or not. They may be overwhelmed when confronted with too much or not enough informaton provided. The main difficulty involves selecting and organizing the information needed to solve the problems.

Mikhal'skii (1975) comments that if a learner does not dwell long enough on the conscious mastery of the conditions in solving the mathematics problems, he/she adapts to a habitual pattern for problem resolution without analyzing or understanding the conditions and questions presented.

Arguments can be forwarded that both normal and special students may encounter similar difficulties in solving nonroutine/nontextbook mathematics word problems but their processes of executing cognitive strategies may influence the quality of task performance.

Many disabled learners appear less active, planful, and organized in task performance than average achieving students. Torgesen and Silver (1979) believed that disabled learners do not utilize the same cognitive strategies in task performance as the normals. This may be due to the fact that they have less knowledge of memory strategies. Reitman (1970) argued that:

> What is measured is not memory processes per se, but the subjects' ability to adapt sufficiently to the demands of a task by developing effective strategies to deal with it. (p. 479)

Torgesen and Licht (1983) suggested that:

> research in the USSR has shown that both L.D. and mentally retarded children do not spontaneously supply their own organization or structure to many tasks to the same extent as normal learners. This has been demonstrated, not only on memory tasks, but also on tasks requiring complex visual analysis and others requiring the formation of verbal instructions. (p. 8)

Wong (1982) explained that disabled learners use a "primitive" organizational strategy and lack metacognitive skills. This is due mainly to their lack of awareness of deliberate control of task appropriate strategies. A similar conclusion was also drawn in Owings, Peterson, Bransford, Morris, and Stein's (1980) research on reading performance between good and disabled readers. The disabled readers' failure to process text as actively and strategically as good readers appears due to the fact that they do not spontaneously monitor their learning strategies. It can be argued that the disabled learners are not consciously aware of what, how, and when to apply strategies efficiently and flexibly in various situations. Wiens (1983) noticed that learning disabled adolescents learn to be passive and do not expect to make things happen for themselves.

Learning disabled adolescents experience difficulties/weaknesses in cognitive processes of attention, concept formation, memory, as well as in verbal and nonverbal thinking, reasoning, and organizational skills. Differences between average achievers and learning disabled in styles of thinking and problem solving have been studied in the reflective and impulsive cognitive styles. Cognitive styles refer to the typical and consistent ways that individuals select and organize environmental data. Learning disabled individuals are characteristically and behaviorally impulsive. They are different from the reflective learners who are able to delay responding while considering and evaluating solution alternatives as well as using more effective strategies for problem solving. Their delay response is strategic in that it is efficient and carefully managed (Cullinan, Epstein & Silver, 1977; Digate, Epstein, Cullinan & Switzky, 1978; Gerber, 1983; Hallahan & Reeve, 1980; Kagan, 1965; Keogh, 1973; Meichenbaum, 1977).

Keogh (1973) has cautioned that cognitive styles only present a specific pattern and organization of cognitive controls; motivation, preference, task demands, sex differences, setting, and instructional programs also contribute to the individual differences in cognitive styles. She comments that some learners may not be strategy deficient but rather strategically inflexible due to the fact that they may overlearn certain preferred strategies.

The most common descriptions of inactive or disorganized cognitive processing cannot explain individual differences within the heterogeneous learning disabled population. Learning disabled individuals are not inefficient in all types of learning situations. There is evidence to support the contention that learning disabled individuals who do not use appropriate strategies are equivalent to the average or above-achievers in knowledge/skills required for task performance (Shepherd, Fleischner & Gettinger, 1979; Torgesen, Murphy & Ivey, 1979). The interaction among individual, task, and environmental characteristics needs to be considered while explaining specific types of deficits, such as poor attention, which may be found in both non-learning disabled and learning disabled individuals. Torgesen (1983) commented that learning disabled students may process information as actively as non-learning disabled, but they either process different information (as when off task) or they may employ less adaptable, but still active strategies. Wong (1982) argued that the learning disabled do not differ substantially in modes of organized strategies from non-learning disabled, although their organized strategies may be less efficient. It was presented earlier in this section that many studies have concluded that the disabled learners would not organize or plan the same cognitive strategies and would experience failure to process information during task performance as actively and strategically as the average achieving students. Furthermore, Hallahan, Hall, Ianna, Kneedler, Lloyd, Loper, and Reeve (1983) believed that the learning disabled were strategy deficient. They also agreed with Torgesen's (1977) conceptualization of L.D. as inactive learners who failed to use task-appropriate strategies. However, most of these studies introduced reading tasks for experimental purposes and selected those subjects functioning at elementary grade levels. An alternate focus of L.D. research can be suggested to explore whether the learning disabled individuals plan and utilize cognitive strategies during task performance similar to the average achieving youngsters, with the exception of qualitative differences.

Lawson (1984) gave an illustration of qualitative differences between experts and novices in employing strategies for mathematical task performance. The term "experts" could mean average achievers whereas "novices" could be construed as mathematical learning disabled for purposes of the following discussion. He suggested that the experts enjoyed a number of cognitive advantages. They have superior knowledge of mathematical content. For example, if a problem requires knowledge of concepts A, B, C and D, the expert will know more about each of these concepts, about how they are interrelated, and about how they relate to other concepts, than will the L.D. A similar notion was supported by Paris and Myer's (1981) study of good and poor readers. Poor readers were less aware of effective strategies as well as being less effective in their monitoring activities during reading.

Lawson (1984) has suggested that the experts have well-established strategies for storing, organizing, and retrieving information. It brings up again the notion of automaticity which explains the difference between "well-established" and "to-be-constructed strategies" discussed by Kirby (1984) and Lawson (1984). Suggestions have been made that:

> an expert is likely to run off the problem solving procedure at a rapid rate, perhaps by employing a well-establshed algorithm that he or she knows to be applicable for this task. So after an initial analysis of the problem type, the expert will access the appropriate content and algorithm and generate the solution. A minimum degree of executive control is needed if this occurs. By way of contrast, the novice may exhibit a much greater quantity of planning, analyzing, monitoring, and modifying activities and still not achieve success. (Lawson, 1984, p. 96)

Simon and Simon's (1978) research of experts and novices solving problems of physics provided evidence that novices have greater frequency of checking and modifying behaviors. An expert may show little sign of strategy control or execution, since their well-established strategies flow with the task performance automatically.

It can be argued that the mathematically learning disabled lack efficiency in satisfying one or more of the defining criteria of mathematics problem solving described by Henderson with Pingry (1953) and Polya (1957) as presented earlier in this section. Some examples of these criteria are related to goal-setting, limitation of habitual pattern of responses, and sensitivity to the problem. They may have similar knowledge and strategies required for task performance as the normals, but the cognitive strategies they apply are inefficient, ineffective, or inflexible

(Shepherd, Fleischner & Gettinger, 1979; Gerber, 1983).

A cognitive view of strategies and task performance

The interrelationship between processes and conceptual knowledge as well as their importance to problem solving are shown in some research findings (Mayer, 1975; Webb, 1979).

Flavell (1979) explained that knowledge means:

> knowing what factors or variables act and interact in what ways to influence the cognitive processes and outcomes. (p. 907)

He further explained that experience refers to any conscious, cognitive or affective experience which stimulates an individual's thoughts and activates the selection of strategies for achieving a goal. Goals/tasks indicate the objectives of a cognitive enterprise, whereas strategies deal with the most effective means employed to achieve them.

Gagné (1983) presented three phases in solving mathematics word problems. The first phase requires the learners to have a certain amount of language competence in order to translate the problem statements into mathematical expressions. They also may recall previous knowledge that is organized into a variety of schemata. Beyond this is the learners' skill in identifying appropriate mathematical operations, such as adding or multiplying. However, planning procedures and the application of mathematical rules are equally significant. The third phase allows the learners to validate a solution by means of carrying out the rules and schemata. The terms "rules" and "schemata" are interpreted as knowledge and strategies being applied during the process of task performance, which is the major emphasis of this study.

Senf's (1971) information-integration approach explains how the individuals selectively attend to units of information which are integrated through their "information array". It is the individuals' internal environment such as their store of memories and mental reactions to the incoming information which influences their actions or behaviors. The theoretical model conceptualizes learning disabilities as involving functional irregularities in the information-processing and information-integrating systems. Some major categories of specific learning disabilities are considered in Senf's theory, such as disabilities arising from the failure to produce or to receive proper information on the array, and disabilities arising from task-irrelevant content occupying the information array. The theoretical model accounts for how an individual acts on the information by assimilating it to his/her past cognitive-experiential repertoire and by referring to the auditory feedback from his/her own vocalization and thought processes evoked by the information.

Solving arithmetic problems, like any other thought process, is an analytic-synthetic process which requires an individual to isolate parts of elements as well as integrate/reformulate relevant information. An analytic process requires searching for a solution that involves breaking down a problem in relation to its known and unknown data. Synthesis is the process of re-establishing the connection/relationship between the analyzed and the known data (Bogolyubov, 1972; Kalmykova, 1975; Polya, 1957).

Thinking and strategies control are the chief determinants for orchestrating analytic-synthetic processes during problem solving. Sternberg (1985) discussed the differences between automatic processing and controlled processes. Their major differences can best be explained in this way: automatic processing requires lower-order nonexecutive processing more closely associated with information retrieved from long-term memory, and controlled processing requires higher-order executive processing involving conscious thinking and planning (Pressley, 1985; Sternberg, 1985). Hudgins (1977) stressed that problem solving demanded productive thinking that called upon an individual to select and organize his knowledge for generating potential solutions. Cawley (1985) suggested that thinking and providing opportunities and time were essential to mathematics proficiency for the learning disabled whereas computational skill was seen as fundamental for application but should not be an end in itself. During the process of problem solving, the individual may reconstruct or redefine what he knows to fit the structures of the given situation.

What influence an individual's cognitive process has on the quality of task performance can best be explained by Flavell's (1979) notion of the dynamic interactions among knowledge, experience, goals/tasks, and strategies. When learners confront a novel task, their existing knowledge concerning the goal will lead them to recall previous experiences from which cognitive strategies are selected, evaluated, and revised while they are doing some reflective thinking/planning. The strategies/actions trigger additional experiences which again are guided by significant knowledge and provoke further searching for cognitive strategies learned/self-generated to see if they provide a means to achieve the goal.

On the other hand, productive thinking is a basic channel that leads to the process of searching for cognitive strategies. Flavell (1979) explained that difficulties might be encountered during the searching process but previous cognitive knowledge and experiences would continue to activate the reconstruction and reassessment of cognitive strategies. Productive thinking also results in an attempt to discover various strategies that can be used in selecting and evaluating alternatives to achieve a solution. Borkowski (1985) suggested that strategy transfer required making a decision upon applying or adapting previously learned strategies for present task demands. To expand Borkowski's (1985) idea requires further illustration of the differences among declarative knowledge (on knowing what), procedural knowledge (on knowing how), and conditional knowledge (on knowing when and why) to apply appropriate strategies for various tasks and situations (Paris, Cross, DeBritto, Jacobs, Oka & Saarnio, 1984). It is believed that the executive process of strategies control influences cognitive search which can be expressed at different levels of efficiency and flexibility and of proficiency and consistency (Cawley, 1985; Blankenship, 1985).

The preceding suggests that an individual's quality of task performance is most likely influenced by knowledge, experience, and strategies. Productive thinking and strategies control deliberately orchestrate the analytic-synthetic process during problem solving.

Descriptive case study approach

A process-oriented but in-depth understanding of individual differences in strategy-monitoring might best be undertaken by the descriptive case study technique. Descriptive case study method here refers to investigating an individual or group of individuals in the absence of experimental controls. It is the naturalistic and uncontrolled characteristics which make the case study method a unique source of information that complements experimental research (Kazdin, 1980). Wolpert (1981) suggested that the case study approach allowed the researchers:

> to uncover not merely the apparent, surface-type phenomena characteristic of the subject, but also the deeper knowledge, skills, values, beliefs and the interrelationships among these factors within the subject. The researchers are seeking answers to how and why questions in addition to determining what and when information. (p. 112)

On the other hand, the case study approach is criticized for its inherent subjectivity and the limited extent of generalizations that can be made from very few subjects. Its subjectivity is apparent in the researcher's predispositions of deciding which behaviors are observed and which ignored, as well as the method of interpreting the data.

Allport (1961) advocated the application of the idiographic approach which allows an intensive study of the individual, as in the case study method. It serves as a supplement to use the nomothetic approach to study groups of subjects. Although the case study approach has its limitations in providing generalizations, it is a useful tool for developing a model or paradigm as well as testable hypotheses.

Gottlieb and Strichart (1981) suggested that a useful way to conduct research was to establish a progression from descriptive to correlational to experimental research. They advocated that descriptive research provides clues for further study as well as for testing the validity of commonly held assumptions regarding the education of disabled learners:

> Descriptive research may be equally informative in the field of learning disabilities. Good descriptive data are needed on the classroom behaviors of learning disabled children, ... the specific type of academic behaviors displayed by children that result in their being referred for categorization as learning disabled rather than reading disabled, and so forth. (p. 139)

Brandt (1972) commented that:

> without sufficient descriptive information, the wrong problems are selected for study, inappropriate hypotheses are tested, and erroneous inferences are made. (p. 15)

The case study approach presents some crucial merits. Its depth and intensive probing characteristics allow the researcher to develop insight into various aspects of human thoughts and behaviors in addition to examining why an individual behaves as he does (Wolpert, 1981).

On the other hand, discrete categories are also applied for this research. The technique of discrete categorization is used for evaluating behaviors that have a clear beginning and end as well as a constant duration. It is different from a frequency measure in that discrete categorization requires listing a number of behaviors and checking off whether they were performed. There is only a limited number of chances to perform the specific behaviors. A frequency measure focuses on the number of times that a particular behavior may occur and the performances are tallied (Kazin, 1981). Discrete categories have been used to assess behaviors in many studies. Neef, Iwata, and Page (1978) applied a checklist to evaluate the effect of training the mentally retarded and physically handicapped youngsters to ride the bus. Different behaviors related to finding the bus, boarding it, and leaving it were included and classified as performed correctly or incorrectly.

To summarize, executive processes and cognitive strategies are suggested as the most essential variables to utilize for exploring the qualitative differences in task performance between the normals and the mathematical learning disabled.

A diversity of approaches have been suggested in defining learning disability. A similar phenomenon is also found in discussing the criteria for studying the characteristics and origins of mathematical learning disability. Arguments are confounded particularly by the controversies of structural and production deficiencies.

Differences in cognitive development between older and younger children include of qualitative differences on memory tasks. Younger children do not always intend to carry out retrieval activities. Information is usually processed superficially and is retained briefly. The cognitive structure of older children is more advanced developmentally; this influences them to process more in-depth information, perhaps semantically, and is thus retained for longer periods of time. Many researchers find that children with learning problems develop slower than the normals in the use of memory strategies, such as verbal rehearsal. However, it is suspected that qualitative differences on task performance would be more likely influenced by individual differences in knowledge, experiences, and strategies available, although the possiblity of developmental differences is not excluded.

Learning disabled individuals are usually considered as inactive, inflexible, and inefficient in task performance. Research focus needs to be upon individual differences in monitoring and executing strategies during task performance. The onus is on the individuals' sensitivity as well as will and desire to perform various tasks which correspond to their "intentional and goal-directed efforts", whereas strategies employed follow the behavior of thinking while carrying out the tasks. At the same time, the individual needs to know when and how strategies act in what ways that will facilitate goal achievement. This follows Vygotsky's (1962) notion that it is conscious and self-reflective awareness of the activity of the mind which influences the how of one's action.

Limited research has been done on mathematical learning disability as well as utilizing a mathematical task as a means for exploring mental processes during task performance. Senf's information-integration approach conceptualizes learning disabilities that involve functional irregularities in the information-processing and information-integration systems. Similar ideas can be found from the Soviet studies on the mentally retarded students' cognitive processes while solving some mathematics word problems. Adapting a habitual pattern for problem solving, such as selecting separate or all numerical data given, is one of their major deficits in task achievement. Solving arithmetic problems, particularly word problems, is an analytic-synthetic process that requires integrating elements/parts of information related to the questions presented. The learning disabled individuals may experience difficulty in information-integration but the chief reason may be more related to their lack of awareness of deliberate control of task appropriate strategies. They may process information as actively as non-learning disabled, but their modes of organizing strategies may be less efficient.

The purpose of this study then was to explore how specific and general strategies were orchestrated during task performance. The descriptive case study approach was also utilized and provided a more in-depth revelation of some qualitative differences of strategy-monitoring between the M.L.D.s and Non-M.L.D.s.

CHAPTER 3

RATIONALE, QUESTIONS, DEFINITIONS

Rationale

Shchedrovitskii (1972) suggested that solving any cognitive problem is a definite mental process. To adequately study mental operations, emphasis needs to be placed upon both process- and product-oriented approaches. This study focuses on the executive processes of cognitive strategies while solving mathematical word problems.

Goodstein (1981) claimed that the term word problem could be used interchangeably with the term verbal problem as defined by 5 major criteria. He suggested that a verbal problem must:

1. use a natural language format to communicate quantitative informaton

2. provide all data or directions for acquiring the data for solving the problem correctly

3. direct the learner to respond to the problem by including a specific question

4. require the learner to comprehend the meaning of the problem to provide the correct answer

5. provide the opportunity to differentiate process from product in the evaluation of performance

A meaningful solution of verbal problems requires the comprehension of the linguistic context as well as selection of appropriate strategies and operation. Computational products of accuracy or inaccuracy do not indicate an individual's mental processes during the task performance. A focus on verbal problem solving requires a shift in emphasis from computation to information processing and decision making (Goodstein, 1981).

It is suggested that strategies represent the operation of cognitive control processes which involve planning, choosing, and monitoring. Learning disabled individuals' failure to execute task-appropriate strategies spontaneously in situations of response uncertainty may be due to their weakness in cognitive flexibility which does not allow strategies to be adaptively applied. Some researchers suggest that executive control processes provide an individual some volitional control over various cognitive routines related to problem solving. Besides, when an individual confronts a task, he/she needs to evaluate the task demands and his/her repertoire of strategies before selecting task-appropriate strategies. This requires deliberate retrieval and skillful integration of specific memories, general knowledge, and cognitive strategies (Brown, 1978; Flavell, 1977, 1979; Flavell, Friedricks & Hoyt, 1970; Torgesen, 1977; Wiens, 1983).

As indicated in the previous literature review, Soviet studies have produced some insight into methods of analyzing the special students' mental processes while solving some mathematical word problems. Mikhal'skii (1975) analyzed the characteristics of the cognitive processes of some mentally retarded students while they solved some mathematics word problems. Four categories of word problems were utilized in their studies:

1. ordinary problems, of average difficulty, familiar to the students from classroom instruction
2. problems with insufficient numerical data
3. problems with superfluous numerical data, and
4. problems without numbers

Some features of their mathematical problem solving processes were identified. Most of them had no difficulty in understanding and solving the ordinary problems that were familiar to them from instruction. Difficulties were found in the other three categories of word problems. Some of them gave all the possible solutions to the problem even though insufficient numerical data were found in the question. Superfluous numerical data was not isolated and they became confused in producing an incorrect solution. Some of them made up numerical data for problems without numbers. They solved the word problems with a habitual pattern of behavior, such as selecting numerical data from separate conditions given without relating them to the questions or striving for an arithmetical operation without realizing missing elements in the problems presented.

It is more meaningful to use different techniques, for example, analyzing verbal protocols and error patterns as applied by the Russian researchers, for examining the individual's mental processes during problem resolution. An approach most commonly found among the Russian cognitive studies is to examine the mental processes according to their objective content and structure which will indicate the solution of particular problems. It does not depend on the subjective methods of different individuals. This study utilized a similar approach which allows an analysis of some crucial preliminary conditions for developing further research on strategy-training and learning transfer.

Schoenfeld (1983) strongly advocated an analysis of strategies executed in solving mathematics problems. He believes that strategic decisions have a major impact on the direction of a solution and on the allocation of one's resources during the problem solving process.

He suggested that:

> there are both objective and subjective components to the framework for analyzing protocols. The objective part consists of identifying, in the protocol, the loci of potential managerial decisions. The subjective part consists of characterizing the nature of the decision-making process at these managerial decision-points and describing the impact of those decisions (or their absence) on the overall problem-solving process. By definition, managerial or strategic action is appropriate whenever a large amount of tactical resource are about to be expended. This provides the basic idea for parsing the protocols: to partition a protocol into macroscopic chunks of consistent behavior (episodes). Then the points between episodes —— at which the direction or nature of the problem solution changes significantly —— are the managerial decision-points at which, at minimum, mamagerial action ought to have been considered (P. 354).

This research advocates that the utilization of a descriptive case study approach to explore strategies execution and control will hold a more significant promise for research in cognitive processes. Detailed discussion of the case study approach was already presented in chapter 2.

Kruteskii (1976) revealed the flavor of his research approach to examining mathematically able adolescents, but similar rationale can be applied to the present cognitive research on M.L.D. subjects:

> It is hard to understand how theory or practice can be enriched by, for instance, the research of Kennedy, who computed, for 130 mathematically gifted adolescents, their scores on different kinds of tests and studied the correlation between them, finding that in some cases it was significant and in others not. The process of solution did not interest the investigator.

> But what rich material could be provided by a study of the process of mathematical thinking in 130 mathematically able adolescents (P. 14)!

Lamon (1972) made a similar point that:

> Work with small groups of subjects or even with one subject at a time, using qualitative methods, should be conducted for the purpose of penetrating the mental activity of the subjects and analyzing mental processes when working exclusively in mathematics (P. 8).

The introspective approach was also applied to collect concurrent verbal data while the subjects were performing the task individually. Reference was mainly selected from Swanson and Watson's (1982) intellectual assessment model but both specific and general probing questions were specially designed for this study. The specific probing questions were geared to tap how and by what means the individual subject went about solving the mathematics word problems whereas the general probing questions would only be presented if the subjects remained silent during task performance.

At the same time, verbal protocols allow an in-depth analysis of concurrent cognitive processes. A protocol is a description of the activities in which a subject engages while performing a task. A coding system of verbal protocols allowed qualitative analyses of sequences of overt actions taken by individuals in the process of solving problems (Ericsson & Simon, 1980; Hayes, 1981; Schoenfeld, 1983; Swanson, 1981).

> "A plan in the heart of a man is like deep water,
> but a man of understanding draws it out"
> (Proverbs 20:5, 1971).

The goal of this study is to analyze what and how specific kinds of strategies are drawn out, during the executive processes, which may influence the quality of task performance found among the individuals.

Moreover, both non-M.L.D. and M.L.D. seventh graders were selected for this study due to the fact that there are many mathematics studies selecting normal students at junior high school levels but very few cognitive studies have chosen adolescents who are learning disabled in mathematics. Emphasis is on the learners' strengths; what they can do on their own.

Two mathematical tasks designed for this study are based upon the reference from Soviet studies as discussed above. Detailed description of the tasks is given in Chapter 4.

Questions to be explored

1. What and how would the cognitive strategies influence the qualitative differences and similarities of task performance found among the M.L.D. and Non-M.L.D. subjects?

2. Would the same cognitive strategies be applicable across the task performance on four different arithmetical operations and the three major categories of mathematics problems?

Definitions

1. Executive process refers to cognitive performance that involves encoding and decoding, retrieval of information as well as the execution of cognitive strategies.

2. Cognitive strategy is an internally organized skill that selects and guides the internal processes involved in defining and solving novel problems. It serves as a means to facilitate the goal-directed task performance across situations and settings.

3. Strategy monitoring indicates the process that an individual uses in checking and changing cognitive strategies utilized.

4. Verbal protocol is a subject's verbal description of his/her mental activities engaged during current task performance.

5. Planning refers to execution and to orchestration of goal-directed actions based upon task demand, environment (perceptual input), repertoire of basic knowledge, and cognitive strategies.

6. Flexibility is defined as knowing how and when to change or continue the direction of task-appropriate strategies for various problem-solving situations.

7. The mathematically learning disabled are learners whose mathematics achievement is 2 years or more

behind relative to chronological age and grade equivalent, despite the fact that they have at least fourth grade or above reading level, an I.Q. of 95 or above, adequate motivation and appropriate educational opportunities with no emotional, neurophysiological, or psychological problems.

8. Analytic-synthetic mental processes are defined as analytic processes requiring a search for a solution which involves breaking down a problem into its known and unknown data. Synthesis is the process of re-establishing the connection/relationship between the analyzed and the known data.

9. Inactive behavior exhibits no deliberate intention to engage in goal-directed task performance.

CHAPTER 4

METHODOLOGY

Subject selection

Two 12- to 13-year-old mathematically learning disabled (M.L.D.) and two average achieving (non-M.L.D.) 7th graders of each sex participated in this study. The subjects came from two junior high schools in the Edmonton Public School System. They were selected from the Cognitive Education Project conducted by Dr. R. F. Mulcahy at the University of Alberta. Parental permissions for the subjects' participation in the study were already obtained through the Project, and students completely both the Canadian Achievement Test and Canadian Cognitive Abilities Test in January, 1985. Their assessment results (See Table 1) were also utilized for this study based upon the following criteria:

1. Both types of subjects had at least fourth grade or above reading level as assessed by the Canadian Achievement Test on reading vocabulary and reading comprehension.

2. The mathematics achievement of the disabled subjects was 2 years or more behind relative to chronological age and grade equivalent, as assessed by the Canadian Achievement Test on mathematics computation as well as mathematics concepts and application.

Table 1: Background information of the 4 subjects

Subjects		Non-M.L.D. 1	Non-M.L.D. 2	M.L.D. 3	M.L.D. 4
Age (years, months)		12.9	13.3	13.1	12.3
C.A.T.					
Reading	(G.E.)	8.2	7	10.4	5.4
Vocabulary	(N.S.,mean=5)	6	5	7	4
Reading	(G.E.)	8.3	6.2	6.8	3.7
Comprehension	(N.S.,mean=5)	5	5	5	3
Math.	(G.E.)	8.1	7.7	5.2	5
Computation	(N.S.,mean=5)	6	5	3	2
Math. concept	(G.E.)	8.7	7.8	6.1	3.5
& Application	(N.S.,mean =5)	6	5	4	2
C.C.A.T.					
Verbal (mean=100,		Unavail-	107	119	112
stanine:mean=5)		able	6	7	7
Nonverbal(mean=100,		Unavail-	111	103	103
stanine:mean=5)		able	6	5	5

G.E. = Grade equivalent N.S. = National stanine

C.A.T. = Canadian Achievement Tests

C.C.A.T. = Canadian Cognitive Abilities Test

Non-M.L.D.: Subject 1 = Male Subject 2 = Female

M.L.D.: Subject 3 = Male Subject 4 = Female

Non-M.L.D. subjects performed at slightly above their grade level in mathematics relative to chronological age as assessed by the Canadian Achievement Test on the two subtests mentioned above.

3. Both types of subjects had an average I.Q. of 95 or above, as assessed by the Canadian Cognitive Abilities Test on verbal and non-verbal batteries.

4. The subjects' current grades in mathematics, as reported on their report cards, were also considered. The two non-M.L.D. subjects' classroom standard in mathematics were B (subject 1) and B+ (subject 2) respectively; whereas both of the M.L.D. subjects had C- in their report cards.

Teachers and the subjects' health records were consulted in order to exclude subjects with any emotional, neurophysiological (including hearing, visual-motor, and communicating functions), or psychological problems. One of the normal subjects (male) transferred from another province after the Canadian Cognitive Abilities Test was administered by the Project. However, a brief interview with his classroom teacher verified his academic and intellectual competence.

Background information for the four subjects is outlined in Table 1.

Pilot studies

Two pilot studies were done in February and March, 1985 at Caledonia Park School. Two subjects were involved in the first pilot, whereas four subjects participated in the second pilot study. All of the subjects involved in both pilot studies were informally assessed to have learning difficulties in mathematics based upon references from their individual school records and results of standardized mathematics achievemnt tests being taken.

Both pilot studies included 12 mathematics problems in three major categories, namely: ordinary problems that are familiar from classroom instructions and are similar to textbook problems; problems with surplus information; and problems with missing information. One surplus or missing element was included in the last two specific categories of nontextbook problems during the first pilot study. It was decided that the level of difficulty could be expanded slightly by means of presenting an appropriate challenge at, or slightly above, the subjects' level of competence. Thus the study was changed to have two surplus/missing elements in both specific categories during the second pilot study. Some general and specific probing questions and a table of cognitive strategies and task performance were utilized for both studies.

The results provided valuable information regarding modifications of the mathematics problems (See Appendix B), probing questions (See Appendix C), and the table of cognitive strategies and task performance (See Appendix D). Detailed explanations of the word problems will be provided in the following section on task. The ordinary problems indicated the subjects' ability to master the four basic arithmetical operations; they could solve similar kinds of textbook problems easily and correctly. Difficulties were encountered by some of the subjects while they were solving the nontextbook problems in the other two categories of mathematics word problems. It was decided that the 12 word problems would be separated into two tasks with two specific arithmetical operations required in each task.

Some of the probing questions introduced into the pilot studies were eliminated due to the fact that they might produce the effect of implicitly framing/leading the subjects' thought processes or attention.

Task

The two major goals of applying mathematics word problems to assess the executive processes of the subjects' strategies are:

1. to utilize the mathematics task as a means to identify the cognitive strategies executed for problem resolution;

2. to examine the applicability of similar cognitive strategies being executed across real-life problems in relation to four different arithmetical operations and three categories of mathematics word problems.

The current focus of interest was on the execution of cognitive strategies in solving three categories of mathematics word problems, namely: ordinary problems; problems with surplus information; and problems with missing information. The first category consisted of problems that were familiar from classroom instruction and thus were similar to textbook problems. The last two categories presented some nontextbook problems which required the subjects to identify with explanations either the extraneous information or the missing information required in order to solve the problems.

The four basic arithmetical operations (+, -, x, ÷) were separated into task A and Task B. Two problems in each category required the application of either addition or division; this constituted a total number of six problems in task A. Two examples of task A are:

1. Mr. Edwards paid $3260 for a stereo set.
   He also spent $457 for buying a rocking chair.
   How much did he pay in all?
   (Ordinary problem requiring addition).

2. Peter picked peaches for 16 days and earned $384 last month. He also earned $95 for picking apricots this month. How much did he earn each day for picking peaches last month?
   (Problem with surplus information requiring division).

On the other hand, two problems in each category requiring the use of either subtraction or multiplication constituted a total number of 6 problems in task B. Two examples of task B are:

3. Kristy has 15 stamp albums. She has 79 U.S. stamps. Each album holds 129 stamps. Two of the stamp albums have 36 Canadian stamps.
   How many stamps does she have in all the albums?
   (Problem with surplus information requiring multiplication).

4. Tom took out $768 from his saving box.
   He bought his sister a birthday gift and also spent $359 for buying a bike last week.
   He bought a pair of skating shoes and paid $27 for 2 records this week. How much did he spend in 2 weeks?
   (Problem with missing information requiring subtraction).

Both tasks provide only whole numbers —— ranging from 2-digit to 4-digit —— and each of the problems required the application of one basic arithmetical operation. Two elements of surplus information or missing information are included in each specific type of nontextbook problem with the exception of problem #4 in task A, and problem #3 in task B. Three surplus elements are built into each of these two problems with the intention of finding out whether there is any difference in the students' ability to realize two or three surplus elements. At the same time, a surplus element ("Two of the stamp albums") in task B, problem #3 was given verbally with a belief that differences may have benn found between realizing surplus elements presented in a numerical and those presented in a verbal context. No extraneous or missing elements are built in for ordinary problems in either task.

Strategy assessment

Protocol analysis

A verbal, or "thinking aloud" protocol involves asking subjects to express verbally all thoughts that come into their minds during task performance (Hayes, 1981).

The "think-aloud" method has been advocated as a rich basis for cognitive assessment. It provides information about the subject's general and specific cognitive strategies as well as any inhibitory or facilitative ideation that interrupts or enhances task performance (Ericsson & Simon, 1980; Garner, 1982; Meichenbaum, 1977; Swanson, 1982).

There are some criticisms with respect to the use of the verbal protocol method for cognitive assessment. Three major criticisms are most commonly found:

1. incompleteness in verbal reports can be found under the condition that unheeded information is not stored in short-term memory;

2. automatic cognitive processing may occurred;

3 not all the information available in short-term memory at the time of the report is actually reported; and

4. the method may handicap subjects with limited linguistic skills, especially younger children.

It has been suggested that if the subjects articulate information directly which is already available to them, then the think-aloud method will not change the course and structure of the cognitive processes; nor will concurrent verbalization under such conditions slow down these processes. It is quite likely that the subjects' verbal protocols may be sketchy, but their mental processes will not be slowed down or changed; if their performances are highly automated; therefore, the subjects may not make much use of short term memory. This phenomenon most likely will be found in acts of realizing familiar tasks or stimuli after long practice. However, the focus of verbal report data mainly gears to explicit metastatements about the process itself replacing statements about inputs and outputs in the protocols. It is also important to stress that the subjects' mental processes may be interfered with only if they are told what to think or talk about (Ericsson & Simon, 1980; Garner, 1982).

Some precautions need to be considered when utilizing the verbal assessment method. The interviewer is encouraged to probe for simple behavior descriptions in a nonspecific, noncuing, "bland" fashion, but the number of probing questions should be minimized in order to avoid interrupting the subjects' cognitive processing or misleading them into making guesses or inferences. Moreover, inquiry must be made as soon as possible in order to assess directly the information in short-term memory (Ericsson & Simon, 1980).

Many approaches are found in the literature relating to protocol analysis (Newell & Simon, 1972; Swanson, 1982; Garner, 1982). Glass, Holyoak & Santa (1979) suggested that:

> the first step in doing a protocol analysis is to break up the transcript into fragments that correspond to different states of the subject's knowledge or mental activity. This division is done intuitively by the experimenter. The result is a series of labelled statements. (p. 413)

Lucas, Branca, Goldberg, Kantowski, Kellogg, and Smith (1979) outlined:

> the definitions for a set of constructs which were to represent observable, disjointed problem solving behaviors and related phenomena . . . . Each event was assigned a symbol, and the collection of events which comprised a problem-solving sequence of processes was recorded in a horizontal string of

> symbols corresponding to the chronological order of appearance during the actual problem solution. In this manner a researcher could listen to a tape of a problem solution (in conjunction with observing written work, interviewer notes, and/or a verbatim transcript) and produce a string of symbols which represented the composite perception of the solution process. (p. 354)

This study utilized both methods mentioned above with some modifications. Analyses broke down each descriptive statements into segments which corresponded to each specific mathematics problem. A table of cognitive strategies and task performance was completed for each subject's verbal expressions of strategies, along with his/her written scripts of each mathematics problem. This allowed the researcher to investigate the kinds of strategies employed and to examine whether the subject's verbalizations corresponded to the written scripts. Two independent tables, with identical format, of cognitive strategies and task performance (completed after finishing protocol analysis were) then compared with the one already made during the subject's actual task performance. Any differences found between the behavioral checklists among the items being checked off was re-evaluated before confirming any final decision.

Error analysis

Error analysis aims to:

> classify errors according to the stimulus properties of the exercises . . . to supplement careful description of the exercises with an

> analysis of the process employed or algorithm used in the incorrect solution. Often it is necessary to interview the learner clinically in order to determine the methods used to arrive at the wrong answer . . . . Error analysis, which examines procedures, often yields data on the status of the learner that will stand in sharp contrast to an analysis based solely on scores. (Goodstein, 1981, p. 33-34)

Error analysis conducted in this study was geared to converge the evidence observed and then recorded in the behavioral checklist of task performance. It also served as an important technique for providing stronger support for the descriptive analysis.

Analyzing computational errors is a common feature of some mathematical studies. Roberts' (1968) model of four error categories —— wrong operation, computational error, defective algorithm, and random responses —— are usually applied in these studies. His model of error categories should not be overstressed because many students' errors may not be simply the result of carelessness, problems with the basic facts/skills, or random responses. Incorrect solutions may also be related to skill/strategic deficit. Suggestions have been made that a careful analysis of task dimensions, such as the operations/computational complexity and reading level, as well as the learner's performance on various types of word problems, can provide diagnostic information of specific skill deficits (Goodstein, 1981).

The nature of this study focused mainly on the cognitive strategies employed during task performance on two specific types of mathematics word problems with missing or surplus elements. The major prerequisites of reading level and basic mathematics skills were already assessed, prior to the screening procedure, at the subjects' current functioning levels. The pilot study and the category of ordinary word problems provided additional evidence of satisfying these two requirements.

Error analysis was done on the verbal protocols, written scripts, and error patterns. Five major error patterns can be elaborated (See Appendix E):

A. Selects part/all of the numerical data, correctly/incorrectly, unrelated to the question

B. Applies all numerical data and/plans incorrectly

C. Selects data needed or related to the question but plans incorrectly

D. Carries out more than 1 step for the main solution

E. Computational error

Some criteria were set up to evaluate the error patterns (See Appendix F) based upon a qualitative and process-oriented approach. The error patterns can be applied to evaluate all mathematics problems in both tasks.

Surplus elements found in task A word problems are:

Problem #2: $95 and 7 more days;

Problem #4: 130 units, 4 people, and 16 units.

Surplus elements found in task B word problems are:

Problem #3: 79 U.S. stamps, two of the stamp albums, and 36 Canadian stamps;

Problem #4: $14 and $110.

Missing elements found in task A word problems are:

Problem #3: the total number and cost of books;

Problem #6: the total number of rows and seats.

Missing elements found in task B word problems are:

Problem #2: the cost of a birthday gift and skating shoes;

Problem #6: the number of push-ups did in this week and the number of days missed in doing it.

Procedures

Two M.L.D. and two non-M.L.D. subjects, each type including one subject from each sex, were randomly assigned to one of the mathematics tasks. Each subject was required to complete an individualized mathematics task consisting of six word problems which were designed according to the three major categories mentioned earlier. Problems found in task A required the application of either addition or division; problems found in task B required the use of either subtraction or multiplication. No instruction was given as to which arithmetical operation should be applied, nor were the subjects notified that surplus or missing elements would be found in the word problems.

However, a brief instruction (See Appendix A), without training/practice, of the think-aloud method, was given by the researcher before the subjects started the task. Subjects were required to include their written scripts for each mathematics problem. They were required to write down the missing elements in short/incompleted sentences after completing their oral explanations or indications. Misspelling and incomplete sentences were accepted if they were relevant to the correct solution. No time limit was set.

The experimenter administered the individual assessment procedures in which some general and specific probing questions (See Appendix C) were introduced, depending upon both situational and individual differences. The subjects' verbalizations were audio-taped individually. The probing questions were designed to draw out the strategies employed by the subjects in solving the word problems.

A behavioral checklist of task performance was completed by the researcher during the actual assessment; that is, while the individual subject was performing the task. No inter-observer reliability was made but intra-observer reliability was carefully applied.
Two additional behavioral checklists (identical in form) were completed by the researcher while analyzing the verbal and written data for each subject at two different time/dates. Only intra-observer agreements among the 3 independent tables (with identical forma) of cognitive strategies and task performance would be recorded for data analysis. Any intra-observer disagreement would be re-evaluated before making any conclusion. However, such incidents were found three times throughout the entire course of data analysis. The behavioral checklist was designed and modified with reference to the models suggested by Days, Kulm, and Wheatley (1979), Kantowski (1977), Webb (1979), and Wheatley (1980).

Each subject required approximately 25-30 minutes to complete the strategy assessment. However, a minimum of two hours was required to transcribe each subject's verbal protocols, and approximately seven hours were spent analyzing each subject's verbal and written data as well as his/her error patterns.

Data analysis

The technique of discrete categorization was utilized to evaluate each subject's data recorded in the table of cognitive strategies and task performance. The cognitive strategies found in the checklist were categorized as performed or not performed while the subject was solving each mathematics problem. Analysis was determined by giving a symbol (1) for each specific strategy performed overtly while solving the six mathematics problems found in each task. A maximum total would be six and a minimum total would be 0 (See Tables 2a, 3a, 4a, and 5a). On the other hand, strategies i, j, k, l, categorized whether surplus or missing information was recognized and indicated with explanations. There was more than one surplus or missing element found in the specific problems designed for both task A and task B (See Appendix B). Some of the subjects recognized or indicated only part of the surplus or missing elements. Results were recorded during the assessment and were re-evaluated while analyzing the verbal and written data; they were also interpreted in descriptive format, even though they were shown on the table. A similar approach was applied for evaluating error patterns.

Verbal protocols were transcribed and analyzed in descriptive format, based upon each word problem, using written scripts and the tables relating to cognitive strategies, task performance, and error patterns. A summary of the four subjects' task performance is shown in Table 6, for which a coding system was devised. Lucas, Branca, Goldberg, Kantowski, Kellogg, and Smith's (1979) idea of developing a coding system was used as the basis for the code, but it was modified for the purpose of this study. Schoenfeld (1983) explained their concepts:

> that particular coding scheme included a two-page "dictionary" of processes that were assigned coding symbols. All behavior was "required to be explicit; otherwise it is not coded" (p. 359). As an example of the coding, the following sequence was coded as R, R, L8 Pi Da5 4: "The problem solver reads the problem, hesitates, rereads part of the problem, says the problem resembles another problem and he will try to use the same method, then deduces correctly a piece of information from one of the given data" (p. 361)" . . . . Kantowske's recent work includes a "coding scheme for heuristic processes of interest" that focuses on five heuristic processes related to planning, four related to memory for similar problems, and seven related to looking back. Researchers explore the frequency of such processes in relation to problem-solving performance". (p. 348-349)

The coding system designed for this study presented each category of strategies, outcomes, and error patterns in alphabetical order. For example, a total of three found in strategy (a) can be interpreted to mean that the strategy of planning was employed while solving three of the mathematics problems. A total of five found in the outcome can be interpreted that five of the verbalizations made in the completed mathematics problems correspond to the written scripts. A total of two found in error pattern (C) can be interpreted to mean that data needed or related to the question were selected but planned incorrectly.

CHAPTER 5

RESULTS AND DISCUSSION

Introduction

Both qualitative differences and similarities in task performance were found among the four subjects. This section will present a descriptive analysis for each subject. The exploratory but qualitative nature of this research was not designed to be correlational, although the four subjects chosen were separated into mathematically learning disabled and non-mathematically learning disabled. The major purpose was to illustrate the qualitative differences and similarities among the subjects in executing strategies during task performance. This study makes no comparison of sex differences in task performance; however, the following discussion will emphasize male and female subjects. The two types of male subjects involved in task A exhibited similar strategic behaviors and error patterns just as both types of female subjects performed task B and also indicated similar strategic behaviors and error patterns. The findings of this study do not necessarily prove that similar strategic behaviors and error patterns will be found in all average-achieving students as well as all mathematically learning disabled individuals.

A table of cognitive strategies and task performance, a table of error patterns, and a summary of the two different tables evaluated for each subject are enclosed individually. Transcribed verbal protocols and written scripts from copies of the originals are found in Appendix G.

Detailed discussions can be found in the final section.

Results

Subject 1 (Male non-M.L.D.)

Task A

Subject 1 was amale non-M.L.D. subject; he was a 12.9-year-old 7th grader. His pre-assessment results in reading and mathematics, measured by C.A.T., indicated that his functioning level in both academic areas was average on the national stanine scale but was equivalent to 8th grade level. His class-standing in mathematics was above average (B). No current I.Q. test result was available but his academic and intellectual competency were verified by his present classroom teacher. The following analysis for subject 1 was based upon the data in Tables 2a (cognitive strategies and task performance) and 2b (error patterns).

Different approaches were used by subject 1 while solving the two ordinary problems (problems #1 and #5). Problem resolution was arrived at spontaneously after reading problem #1. Planning, restating the problem in his

Table 2a: Cognitive strategies of task A performance

Subject 1

Problems	1	2	3	4	5	6	Total
Strategies							
a. Indicates planning		1	1	1	1	1	5
b. Rereads the problem		1	1	1		1	4
c. Restates the problem in own words			1	1	1		3
d. Draws a diagram							
e. Underlines information given					1		1
f. Makes systematic trial-and-error					1		1
g. Makes random trial-and-error							
h. Makes hypothesis			1			1	2
i. Recognizes surplus information		2		3			5
j. Indicates surplus information		2		3			5
k. Recognizes missing information						2	2
l. Indicates missing information						2	2
m. Checks/corrects step/s of solution							
Outcome							
Verbalizations correspond to the written scripts	1	1	1	1	1	1	6

Table 2b: Error patterns

Subject 1

Error Patterns	1	2	3	4	5	6	Total
A. Selects parts/all of the numerical data, correctly/incorrectly unrelated to the question.							
B. Applies all numerical data and/plans incorrectly.							
C. Selects data needed/related to the question but plans incorrectly.			1				1
D. Carries out more than 1-step for the main solution.							
E. Computational error.				1			1

own words, and making systematic trial-and-error strategies were applied in solving problem #5. He restated the problem in his own words by saying: "There are 735 notebooks and there's 15 boxes. The question is how many notebooks are in each box." A planning strategy was proposed when he stated: "Gonna to divide 15 boxes into 735 notebooks and it'll give you the answer of how many notebooks are in each box." He tried out the strategy of systematic trial-and-error five times in terms of using multiplication steps to find out the product of the division problem. His explanation was: "Because I can't really tell myself what's how many going to that. So, I want to be sure. So, I want to take all these numbers like 4, 5, 7, 8, 9 . . . and see which one that's exactly the number." A correct answer was achieved, but the quotient of the divisor usually could be found by the normal individuals at his age and grade level using calculation rather than trial-and-error. The strategy of systematic trial-and-error was executed less efficient, both in terms of time and effort required.

Rereading and planning strategies were utilized while he was solving problem #2. He decided on a mathematical operation with quick planning after rereading the problem: "So, you'll divide 384 by 16. Yap, I think that's what I'll do." He indicated all the surplus elements with explanations and comments such as: " I don't need to know how many apricots he's picking or anything or how much money that he earned cause the question is how much he earned from peaches. So that means I don't need all that information. It's just trying to throw in some things. Try to trick you."

The protocols indicate that he was quite aware of choosing the information needed to answer the question. Evidence of strategies utilized for information-integration was found in his verbal explanations of chosen information: "O.K. It says Peter picked . . . last month. Then, you read the other information. It says he also earned 95 dollars . . . . How much did he earn for picking peaches last month? So, you don't need the information of apricots because it said nothing in the question about apricots. So, you go up to the first line. You read it over and then you see that it says about peaches. Then, it says Peter picked . . . 384 dollars last month. That means how much he earned each day for picking peaches last month. So, the information that's in the middle here about the apricots you don't need because there's nothing saying apricot in that question".

The strategies of attending to selective information on peaches but ignoring the surplus information on apricots were also seen when he was indicating the surplus elements related to apricots: "Well, it's how much it's saying that in 7 more days he spent on picking apricots that month and earned 95 dollars for picking apricots. You don't need to know that . . . . So, why bother looking at that information, when you know this is all you need."

None of the missing elements found in problem #3 was realized. Hypothetical thought and correct operation were evidenced but wrong assumptions (inappropriate hypotheses) led to incorrect solutions: "Well, it says here 9 of the books and just says some books. I presume that 9 of the books is all the books. So, 16 records, 16 records. Your information is right there together for the books and records. Then, you only got your poster left and 7 posters cost 24 dollars. Whatever you get would be how much would it cost for all the records, books, and posters. So, you add these up and you'll get the answer, 294 dollars." He confirmed that he had all the information needed to solve the problems: "Yeah! You have all the information because it gives you 2 of your products or things into one price. Then, you've only got three products and your other product and the price is right there. So, you have all the products together." He presumed that the cost of books and records was given in one price; this resulted in an incorrect solution.

However, a good example of analytic-synthetic behavior was seen while he was explaining the information applied for problem resolution. Evidence of his analysis was illustrated in his verbal protocols: "I used my title that starts the problem which is a school library . . . and

7 posters. So, I know when it says 9 of the books and 16 records cost 270 dollars." Evidence of synthesizing needed information was also illustrated in his verbal protocols: "Well, I go back to the top lines. It says 16 records. So, I know there's no records left. It says 9 of the books and then it says some books. It says 7 posters cost 24 dollars. So, I go back up. It says 7 posters, so I know there's no one left. I just add those up and get the price that'll be how much it all cost." Different pieces of information were decomposed during his analysis, but they were also integrated according to their numerical and verbal features (key words). Evidence of integrating information can be found when he mentioned he went back twice to the related data that was given twice.

Rereading, restating the problem in his own words, and planning strategies were applied while solving problem #4. All 3 elements of surplus information were realized; although a computational error related to regrouping was made, resulting in an incorrect answer.

All 3 strategies, as mentioned above, were indicated as soon as he began to approach the problem by means of rereading and restating it in his own words. His verbal protocols provided such evidence: "So, that means you have got 826 people living in 130 units. The question is how many people will be found in the building . . . . Really all you need is to find out how many people living in that building altogether. It says 147 more people are coming. So, you add those together and that'll be how many people will be found in the building." Analytic-synthetic mental processes were evidenced when he indicated with explanations the surplus and needed information in relating to the question: "I don't need to know the information of so many people are living in 130 units, but I do need to know how many people that're living in 130 units. I don't need to know how many people are living in each unit. I don't need to know how many units on each floor. I don't need all that stuff. I just need to know how many people live in that building right at that time . . . . It says a building has 826 people living in right at this time. Then, it tells you there's 147 more people coming. The question is how many people will be found in this building. That means how many people are gonna be starting to live in this place. So, you add 826 and 147, and get your answer."

He also realized that a similar problem with surplus elements (problem #2) was encountered earlier. Evidence was found when he made a comment: "Then, it says 147 more people will move in. How many people will be found in the building. Well, all this information, that the stuff again that you might not even need because it's saying that so many people living in 130 units."

He chose a correct arithmetical operation but made an error in regrouping, and so produced an incorrect solution. This was illustrated in his written scripts as follows: 826+147=983.

Rereading, underlining information given, making hypotheses, and planning strategies were used while solving problem #6; this resulted in a correct solution.

He reread the entire problem after finishing his first reading aloud. Missing elements were indicated spontaneously and correctly, while the application of an analytic-synthetic method facilitated the accuracy of problem resolution. Evidence was found in his verbal protocols; he reread the entire problem aloud before stating that: "Many rows of seats are found in the music hall. O.K., so that tells me that in this music hall, a lot or many rows of seats are found. 685 people . . . . Well, I haven't got enough information to tell me. Cause it just says many rows of seats . . . and then, it just says 685 people filled up some of the seats." He underlined the word "people" but made the comment: "I don't need to know how many people filled up some seats."

Planning strategy was indicated as he stated the information he was looking for: "I want to find out how many seats are found in one row, or in such and such amount of rows. Then, I was gonna to find out how many seats are to each row. I haven't got any idea or information. It just says many rows of seats. I don't know how many is many really. Could be 6, 7." Evidence of making an hypothesis and executing planning strategies was found in his assumptions for problem resolution: "What I need to know is, maybe how many seats are found in each row. Maybe they can give me there're 7 rows and 42 chairs were altogether. How many seats are found in each row? Well, you maybe divide 7 into 42. Maybe 6 to each row."

Rereading, restating the problem in own words, making hypotheses, and planning strategies were found most distinctively while solving all the word problems. His unique style of choosing needed information in relating to the question, as well as utilizing analytic-synthetic methods in planning and executing strategies, spontaneously facilitated problem solution.

Subject 2 (Female non-M.L.D.)

Task B

Subject 2 was a female non-M.L.D. subject; she was a 13.3-year-old 7th grader. Her pre-assessment results in reading and mathematics, measured by C.A.T., indicated that she was at the average level (5) in both subject areas according to the national stanine scale but was equivalent to 7th grade in reading and 7.8 grade in mathematics concepts and applications. Her class-standing in mathematics was above average (B+) and her I.Q. scores, measured by C.C.A.T., were above average in both verbal stanine (6) and non-verbal stanine (6). The following analysis was based upon the data in Tables 3a (cognitive strategies and task performance) and 3b (error patterns).

Subject 2 could only solve the two ordinary problems correctly. She did not notice any surplus or missing elements designed for four of the problems.

She could solve the two ordinary problems (problems #1 and #5) quickly and accurately. Her choice of mathematical operation was related to the question presented in problem #5 as found in her verbal protocols: "Cause it asked how much she had left after buying a T.V." A planning strategy was used while solving problem #2, but it did not guarantee a correct solution.

Table 3a: Cognitive strategies of task B performance

Subject 2

Problems	1	2	3	4	5	6	Total
Strategies							
a. Indicates planning		1	1	1		1	4
b. Rereads the problem						1	1
c. Restates the problem in own words			1			1	2
d. Draws a diagram							
e. Underlines information given							
f. Makes systematic trial-and-error			1				1
g. Makes random trial-and-error							
h. Makes hypothesis						1	1
i. Recognizes surplus information							
j. Indicates surplus information							
k. Recognizes missing information							
l. Indicates missing information							
m. Checks/corrects step/s of solution							
Outcome							
Verbalizations correspond to the written scripts	1				1	1	3

Table 3b: Error patterns

Subject 2

Error Patterns	Problems 1	2	3	4	5	6	Total
A. Selects parts/all of the numerical data, correctly/incorrectly unrelated to the question.		1	1	1			3
B. Applies all numerical data and/plans incorrectly.				1			1
C. Selects data needed/related to the question but plans incorrectly.						1	1
D. Carries out more than 1-step for the main solution.		1	1	1			3
E. Computational error.							

She planned a two-step operation for problem resolution without noticing any missing element. Related evidence was found in her oral plan: "I do subtract, adding to that each of the numbers . . . . How much he spent in 2 weeks. O.K., he took out 768 dollars. Then, he spent 359 dollars plus he spent 27 dollars . . . . You add the 9 and 7, 16 . . . 386. Subtract from 768 . . . . He spent 382 dollars . . . I used what he spent for the bike, the records, and skating shoes. I subtracted it from the amount he took out. Add those two together." Her oral plan also indicated that verbalizations did not correspond with her written scripts in procedures of operation applied. She actually did addition before carrying out subtraction; this was shown in her written scripts, which were different from her oral plan. Other similar evidence was found when she stated that: "He spent 359 dollars for his bike and 27 dollars for 2 records." She actually did not use any information relating to the skating shoes; she did not realize this information was a missing element. It was contrary to her oral plan that she used what was already spent for the skating shoes.

She selected all of the numerical data incorrectly since it was unrelated to the question. A habitual pattern of arithmetical operation was followed. She did not realize the question asked the amount of money spent, but not how much of it was left; the inappropriate arithmetical operation was indicated in her written answer. Another interesting piece of evidence indicated that she did not attend to the problem or question deliberately, nor was she involved in any reflective thinking. When the researcher asked her why she would add 359 to 27, she replied: "Cause to get the total of what he spent that way you can subtract from the total he took out from his saving box. I took 768 dollars and I subtracted the total which is 386 and I got 382 dollars that he spent in 2 weeks." The researcher continued to ask her why she would do subtraction. She actually stated the correct answer, the product of her first step of the operation, although she did not realize her error: "Because he spent 386 dollars in 2 weeks. The total is 768 which he took out from his saving box." An incorrect solution was the result.

Restating the problem in her own words, making systematic trial-and-error, and planning strategies were applied while solving problem #3. She did not realize there were any surplus elements but applied all the numerical data and carried out more than one step for the main solution; this resulted in an incorrect solution.

Evidence of the three strategies used was found in her verbal protocols when she restated the problem in her own words at the beginning: "How many stamps does she have in all the albums? O.K., Kristy has 15 albums. 79 U.S. and she's got 36 Canadian stamps. You add 79 and 36 together. She's got 115 stamps and divide that into. Divide 15 into 115. 15 times 3 equal . . . (3 different systematic trial-and-error), so I go 7 . . . put a decimal. You go 15 into 6 . . . . She has 7.6 or estimate of 8 stamps in each album."

She did not recognize the surplus elements but used two of the numerical features, and also did not realize a third surplus verbal element. This was indicated in her oral protocols: "I used that she had 79 stamps from U.S. and I added that with 36 Canadian stamps, and I got 115 stamps altogether. Then divided 15 albums into 115 stamps and I got 7.6 or an estimate of 8 in each album." Her verbal protocols also indicated that she did not attend to the question deliberately when she stated that the answer was 8 stamps in each album. When the researcher asked her why she would do division, she actually stated the question correctly but did not realize the error she made earlier: "Cause they want to know the question how many stamps she has in all the albums."

Planning strategy was applied while solving problem #4, but she applied all surplus elements, planned incorrectly, and carried out more than one step for the main solution; an incorrect solution was arrived at.

Planning strategy was found. She stated: "He paid 748 dollars total. I'm trying to find out how much he paid for the housing rent. I take 14, 110, and 139 dollars. Then add together . . . 268 dollars and subtract from 748 dollars . . . . He paid 485 dollars for housing rent."

Surplus elements were not noticed but were applied for problem resolution; this was shown in both of her written scripts and verbal protocols: "I used that he had 768. He paid total 748 dollars for his phone bill, housing rent, and grocery bill. It told how much he paid for the monthly rent of his phone and for 110 dollars for painting the door. For the phone bill and the grocery bill added up to 139. I added 110, 14, 139 up and I got 263. Then I subtracted that from 748 which was the total. I got 485."

On the other hand, verbalizations which did not correspond to her written scripts were found in two instances:

1. She stated in her oral plan: "I take 14, 110, and 139 dollars. Then add together . . . 268 dollars . . . ." She actually wrote down 263 as shown in her written scripts.

2. Her oral explanations of information used: "I used that he had 768. He paid 748 dollars for his phone bill . . . ." Only 748 was given in the problem; it was also in her written scripts. It was considered as deviating from her initial statement.

Rereading and restating the problem in her own words, making hypotheses, and planning strategies were utilized while solving problem #6, but none of the missing information was noticed. She selected data related to the question, but she made an inappropriate hypothesis and misinterpreted the problem; this resulted in error.

Evidence of applying the 4 strategies and the error she made were found in her oral plan: "O.K. 12 push-ups each day every week. A few means 3. So 15 times 3 . . . is 45. He had to do 45 push-ups for this 3 days that he missed." She made an inappropriate hypothesis that a few meant 3 and did not realize any missing elements. An incorrect plan for the solution step resulted in error.

The three most commonly found error patterns while solving the 6 questions were: 1. selecting parts/all of the numerical data and carrying out more than 1 step for the main solution. Both types of error were found in problem #2 with missing information, as well as problems #3 and #4 with surplus elements. 2. Applying all numerical data and/or planning incorrectly was found in problem #4 with surplus elements. 3. Selecting data needed/related to the question but planning incorrectly was found while solving problem #6 with missing elements. Planning strategy was most commonly used while solving problems #2, #3, #4, and #6 but it did not guarantee accurate problem resolution.

Subject 3 (Male M.L.D.)

Task A

Subject 3 wasthe male M.L.D. subject; he was a 13.1-year-old 7th grader. His pre-assessment results in reading, measured by C.A.T., showed that his achievement level in reading vocabulary was above average (7 in national stanine) and average in reading comprehension (5 in national stanine). His achievement level in mathematics, also measured by C.A.T., was two years behind relative to grade equivalent and was below average according to the national stanine scale (3 in mathematics computation and 4 in mathematics concepts and applications). His class standing in mathematics was also below average (C-), whereas his I.Q. scores, measured by C.C.A.T., were above average (7) in verbal stanine and average (5) in non-verbal stanine. The following analysis for subject 3 is based upon the data shown in Tables 4a (cognitive strategies and task performance) and 4b (error patterns).

Subject 3 chose an arithmetical operation spontaneously relating to the question for each ordinary problem (problems #1 and #5). His explanations for using addition for solving problem #1 provide such evidence. For example, in solving problem #1, he stated: "To add the 3260 to the 457. To see how much he paid in all." He explained

Table 4a: Cognitive strategies of task A performance

Subject 3

Problems	1	2	3	4	5	6	Total
Strategies							
a. Indicates planning		1	1			1	3
b. Rereads the problem		1				1	2
c. Restates the problem in own words							
d. Draws a diagram						1	1
e. Underlines information given							
f. Makes systematic trial-and-error					1		1
g. Makes random trial-and-error							
h. Makes hypothesis			1			1	2
i. Recognizes surplus information		2		2			4
j. Indicates surplus information		2		2			4
k. Recognizes missing information			2			1	3
l. Indicates missing information			2			1	3
m. Checks/corrects step/s of solution							
Outcome							
Verbalizations correspond to the written scripts	1	1	1	1		1	5

Table 4b: Error patterns

Subject 3

Error Patterns	Problems						
	1	2	3	4	5	6	Total
A. Selects parts/all of the numerical data, correctly/incorrectly unrelated to the question.		1					1
B. Applies all numerical data and/plans incorrectly.							
C. Selects data needed/ related to the question but plans incorrectly.						1	1
D. Carries out more than 1-step for the main solution.							
E. Computational error.							

that the answer to item #5 could tell him the number of notebooks found in each box. Systematic trial-and-error was utilized while solving problem #5: "I take 15 divided by 735 . . . times 15 times 8 . . . 120 [1st systematic trial-and-error], nine! [2nd systematic trial-and-error]" He said that he took 15 divided by 735 but 735 divided by 15 were indicated in his written script. His verbalizations deviated from his written script.

Rereading and planning strategies were used while solving problem #2. Recognizing and indicating verbally the two surplus elements did not guarantee correct solution, although he could select the information related to the question. Evidence was found in his verbal protocols that he involved himself in reflective thinking by remaining silent after he finished reading the problem once. When the researcher asked what he was thinking, he said: "I'm thinking . . . I'm not sure, ha! But . . . ." Then he remained silent before continuing: "I'm trying to figure out how much money he made for picking peaches for 16 days and earned 384 dollars last month. That's the answer to the question. Right?" He interpreted the question asking the amount of money earned in 16 days for picking peaches but did not attend to the words "each day" presented in the question. Further evidence was found when the researcher continued to ask him what he thought. He reread the problem: ". . . How much did he earn each day for picking peaches last month. Would be 384 dollars." He focused on the words "peaches" and "last month" which were sounded out at a higher pitch and longer duration. When the researcher asked him why he would write down 384 as the answer, he confirmed that it told him the amount of money made for picking peaches last month without specifying "each day".

He could indicate with correct explanations that the 2 surplus elements relating to apricots were not needed to solve the problem, such as: "Cause the question never asked you to use it . . . . The information that he earned 95 dollars for apricots and decided to spend 7 more days this month. So, that shouldn't be in there at all." However, he selected part of the numerical information given but ignored a needed element (16 days); this resulted in an incorrect solution.

A planning strategy was executed spontaneously when he finished reading problem #3 orally once and paused in silence briefly.

He noticed the two missing elements and commented: "Well, actually I couldn't find out the answer to that question . . . . Cause it says some books and then it says 9 of the books and 16 records. So, I've got to find out how many books it had and how much the books cost." Plan development was also shown by his last statement about the needed information for which he was looking. Similar evidence was found when he stated: "I've got to add the posters to the records, to the books. But I don't know how many books there are . . . . Cause you need more information." He initially said that he needed to find out: "How many books and how much would they cost . . . . How much does one book cost or all of them." He further explained that the problem could be solved if the two missing elements were given by means of making an hypothesis: "Say, there's 5 books and 13 dollars each. Then, I would add . . . multiply 13 times 5 and plus that . . . . I add that to the 9 books and 16 records, and add it to the 7 posters." He assumed that each book had the same price and thought that the product of 13x5 could be added to the cost of 9 books and 16 records. If the cost of all/each of the 5 books was found, then the cost for 9 of the books would become unneeded information.

His assumptions would be questionable in such an instance. It was interesting to find out that he integrated the partially provided data with the question while he was looking for the missing elements: ". . . The school library ordered 16 records . . . cost 270 dollars. But, I don't know how many books there were, O.K. Then, it says how much would it cost for all the records, books, and posters." A correct solution was reached.

He decided on an arithmetical operation immediately and accurately after reading problem #4, resulting in a correct solution. He could indicate the two surplus elements in relating to the question by stating the information he did not use: "An average of 4 people live in each unit and 16 units on each floor . . . . Cause the question says how many people will be found in the building, not how many units or how many family, but how many people." Another surplus element (130 units) was not recognized.

His initial explanation of adding 826 and 147 people to solve the problem was: "The way to put them together that would make sense. Wouldn't minus them or divide them. You plus them." When the researcher asked him why he would think that he needed to do addition, he responded: "Why not?" The researcher sought more in-depth analysis by asking him 3 more questions in chronological order:

1. "What would this answer tell you?"

He replied: "It'll tell me how many people are living in the building."

2. "How did you find it out?"

He replied: "By adding. By adding 826 people to 147 people."

3. "Why would you add them up?"

He replied: "To get the total of 973 people."

As table 4a indicates, rereading, drawing a diagram, making an hypothesis, and planning strategies were utilized while solving problem #6. He did not recognize one of the missing elements, and made inappropriate hypotheses for problem resolution.

He realized that the problem could not be solved and explained: "You need to know how many seats there are. Which seats that people filled." He wrote down the information required to solve the problem: proportion of seats filled, how many seats, size of building, size of seats, and space between rows. He was required to explain the meaning of "where're seats filled" that he wrote down: "Like, there's 30 rows, you have to know how many in each row, 15, 30, 100 . . . O.K., and how many seats and also the size of building, and size of seats." He could not tell why it would be required to know where the seats were filled to solve the problem. He erased the sentence and said: "I thought of something out of wasn't something. I thought of something relevant but wouldn't relevant . . . . Like, when I got the size of seats, I know how many there were . . . .

You don't need it. It's just extra information." His change of plan did not indicate another missing element. The researcher asked him why he would need to know the number of seats (as shown in his written scripts) in order to solve the problem. He replied: "Because I would know how many rows are and the size of the building. Oh! I also have to . . . . Space and rows or between rows . . . . O.K., I have to know the size of the seats. To see how many seats will fit into the building.
I have to see the space between the rows or the seats." He actually could not indicate the missing element relating to the total number of rows, although one of his hypotheses indicated in his oral plan focused on the space between the rows.

He drew a diagram while explaining how to solve the problem if all the pieces of information were given. He explained that information regarding space between rows was needed because: "Say, the space is 500 meters, could be no row. Like, could be any number. Could be just a foot and fit more chairs in. The more space between them, the less chairs you can fit in." He made an excellent attempt to look for the missing information but inappropriate hypotheses did not lead him to discover one of the missing elements.

With this subject, rereading, making hypotheses, and planning strategies were most commonly used in solving the six problems. The major errors found were related to: 1. selecting only part of the required numerical data as found in problem #2 with surplus elements, and 2. selecting data unrelated to the question and making inappropriate hypotheses during planning for problem resolution; for example, in problem #6 with missing elements.

Subject 4 (Female M.L.D.)

Task B

Subject 4 was the female M.L.D.; she was a 12.3-year-old 7th grader. Her pre-assessment results indicated her achievement levels in reading, measured by C.A.T., were below average (5) in both reading vocabulary (4 in national stanine) and reading comprehension (3 in national stanine) but were equivalent to 5.4 grade level in reading vocabulary and 3.7 in reading comprehension. At the same time, her pre-assessment results in mathematics, also measured by C.A.T., were below average in both mathematics computation (2) and mathematics concepts and applications (2), and equivalent to 5th grade in mathematics computation and 3.5 grade in mathematics concepts and applications. Her class standing in mathematics was below average (C-), whereas her I.Q. scores, measured by C.C.A.T., were above average in both verbal stanine (112) and non-verbal stanine (103). The following analysis is based upon the data indicated in Tables 5a (cognitive strategies and task performance) and 5b (error patterns).

Subject 4 could solve the two ordinary problems (problems #1 and #5) quickly and accurately, although a routine solution and application of an arithmetical operation were indicated in her verbal protocols.

Table 5a: Cognitive strategies of task B performance

Subject 4

Problems	1	2	3	4	5	6	Total
Strategies							
a. Indicates planning	1	1	1	1			4
b. Rereads the problem			1				1
c. Restates the problem in own words	1						1
d. Draws a diagram							
e. Underlines information given							
f. Makes systematic trial-and-error		1		1			2
g. Makes random trial-and-error		1					1
h. Makes hypothesis							
i. Recognizes surplus information				1			1
j. Indicates surplus information				1			1
k. Recognizes missing information							
l. Indicates missing information							
m. Checks/corrects step/s of solution							
Outcome							
Verbalizations correspond to the written scripts	1			1	1	1	4

Table 5b: Error patterns

Subject 4

Error Patterns	1	2	3	4	5	6	Total
A. Selects parts/all of the numerical data, correctly/incorrectly unrelated to the question.		1	1	1			3
B. Applies all numerical data and/plans incorrectly.		1					1
C. Selects data needed/ related to the question but plans incorrectly.			1	1		1	3
D. Carries out more than 1-step for the main solution.		1	1	1			3
E. Computational error.							

(Columns 1–6 are headed "Problems".)

For example, planning strategy was indicated while she was restating problem #1 in her own words and writing down the numerical data given. However, her explanations for choosing multiplication to solve the problem were based upon her belief that: 1. the two given numbers are two main factors, 2. they are large numbers, and 3. they have to even out. There was an indication of mechanically using a rote arithmetical operation without sufficient understanding of the conditions of the problem. Some algebraic terms were found in her verbal protocols, such as: "main factors" and "even out" although they were unrelated and not applicable to the question asked.

As is evident from Table 5a, planning strategy, and random and systematic trial-and-error were utilized but missing elements were not noticed while solving problem #2. Three whole numbers with similar numerical features ($768, $359, $27) were chosen. She decided to carry out a one-step addition immediately after reading the problem orally. An incorrect answer was found at the beginning due to her selection of all numerical data without recognizing any missing elements. On the other hand, she indicated that she had all the information needed to solve the problem, but her oral protocols did not correspond to her written scripts in terms of explaining her choice of one piece of the numerical data: "I used how much he took out [$768 was given in the problem]. Oh! I did not, and how much he spent on his sister and how much he spent on his bike and how much he paid for 2 records." She actually had used $768 to do addition in her written solution.

Both the arithmetical operation and plan were changed after her initial trial. When the researcher asked her what she did with the chosen information, she commented that she shouldn't add the 3 numerical data together. Her explanations were: "First, you add them to find out how much the total was . . . the two here he spent. I added the one that he took out from his saving box and I wasn't supposed to do that. You are supposed to divide by them how much he took out from his saving box. I choose 386 and 768." All numerical data applied were unrelated to the question and planned incorrectly; this was shown in her initial written script. Her change of plan and arithmetical operation did not shift to produce a correct solution, although systematic trial-and-error in terms of using multiplication steps to find out the product for the division was found in her written script. The product was 1.989 in her written script. The same answer was copied on the answer sheet, but she erased the decimal and changed the product to 198.9 while verbalizing: "He spent a hundred and ninety-eight and ninety cents." Her verbalizations of the product did not correspond to her written answer indicated on the answer sheet. Although she explained that her choice of using division in doing the last step of solution aimed to find out the amount of money spent out of 768, the problem was actually misinterpreted, as if it was asking for the amount of money left. Besides, division should not be used even if the question asked for the amount left. Her second plan also indicated strove for all possible operations without recognizing the missing elements; this resulted in an incorrect solution.

Rereading and planning strategies were applied in solving problem #3 but they did not guarantee a correct solution. Two major error patterns were found; her incorrect selection of all numerical data, (some of which was unrelated to the question); and she used more than a one-step solution. Two of the surplus elements (Two and 36 given in verbal and numerical features separately) were not recognized.

She initially wrote down 15. Then she reread the first 3 sentences of the problem before writing down 129x17 (15 was given in the problem but not 17). When the researcher asked if she used all the information given, she said: "Yes. No, I didn't. I didn't use she has 79 U.S. stamps and 36 Canadian stamps . . . . Because it wasn't needed. There are 15 stamp albums. 17 of them have 129 stamps in it, and 2 of them have 36. The question says how much does she have in all the albums and that's the answer." The researcher continued to ask her what she did with the information she chose. She explained: "O.K. I did 13x129 and 36x2 and I added those together." She was asked to indicate the information she chose: "I chose 15 stamp albums, 129 stamps, 2 stamp albums with 36 stamps . . . . You have to find out how many there are in the 13 and how many are in those two and then you add them together to find the answer." She actually used two surplus elements (Two and 36) in doing multiplication; this was shown in both of her verbal protocols and written script; even though in her initial explanation of the information chosen she said that she did not use the 36 Canadian stamps. Thus, she selected numerical data unrelated to the question. She also understood that there were 17 stamp albums. She appeared confused by two surplus elements (Two and 36), as expressed in her verbal explanation by means of adding them to the 15 stamp albums given.

Her oral protocols did not correspond to her written scripts in relation to one piece of the numerical data given (15 stamp albums). She wrote down 15, 17, and then 129x17 in her written script, which deviated from her oral protocols. She mentioned 15, 17, and 13 stamp albums in various statements of her oral protocols. The researcher decided to ask her again what she did with the information she chose for verifying such evidence. She replied: "O.K. I took 13 and 129 and times them and I times 36 times 2." Researcher continued to ask: "Can you tell me the first step that you did with the information you chose?" She said: "129 times 17 equals 1903 and then I took that and I times, times, um . . . 129 that I wanted and I got 1290. Then, I added those together, and I got 2193 and then I added 72 equals 2265." She also explained that the reason for addition in the last step of solution was because the question was asking the total number of stamps in all the albums. Actually, only the information that there were 15 stamp albums and that each album holds 129 stamps was needed to solve this multiplication problem. Carrying out more than a one-step solution due to incorrect data selection and planning led to an incorrect solution.

In solving problem #4, she used the strategies of systematic trial-and-error as well as planning but she did not recognize a surplus element; this resulted in an incorrect solution.

She chose two given numbers (748, 139) and decided to use division to solve the problem after reading it orally only once. Systematic trial-and-error was used 4 times when applying multiplication steps, according to her written script, to find out the product of division. When she verbalized the answer, she immediately realized that it was incorrect: "So, 53 dollars and 19 cents. No, ha! O.K. . . .

I found out that you can minus it instead of dividing it. Because if you paid 748 and half of it was for his grocery bill, then you can just minus it." Her oral explanations of information used were: "I used 139 and 748 . . . and I divided it." When she was asked the reason for doing division, she said: "You don't. You're not supposed to . . . . I did it wrong. Well, I thought so because of the way that it's written out that it mixed me up. Now, you have to add the monthly rent for his phone and for his grocery bill together (she had 139+14=153 in her written script) and then you minus it from 748 . . . ." She also noticed a surplus element ($110 for painting) that was not needed for the solution: "Because he paid 748 for his phone bill, housing rent, and grocery bill. The question asked how much did he pay for just his housing rent. It tells you how much he paid for his grocery bill and phone bill but not the housing rent . . . . I used the 748 and the 139 for his grocery bill and phone bill." Her procedures of utilizing the chosen information were: "I added 139 and 14 together and I got 153 . . . . It tells me how much he paid for his phone bill and grocery bill. Then, I subtract that from 748 and I got the number that he paid for his housing rent." Her change of plan did not lead to correct problem resolution. The difficulty was due mainly to her selection of a surplus element ($14) unrelated to the question. She also chose the data needed ($748, $139) for solving the problem although both of her plans were incorrect.

She wrote down 15x5 immediately after reading problem #6. Missing elements were not recognized. She confirmed that she had all the information needed to solve the problem. She also provided an explanation for the information chosen: "How many he did, he did, the day of this week and how long he stopped doing them for . . . . Because O.K., there're 7 days in a week and he stopped doing it for 2 days. So, that would be 5 days left and you times 15 times 5 and that's how many he did this week." She chose the data (more than 15 push-ups) related to the question but misinterpreted "a few days" as 5 days without recognizing the missing elements; this resulted in an incorrect solution.

Planning strategy was applied while solving four of the problems, but it did not guarantee a successful solution for most of them. Only the two ordinary problems (#1 & #5) were solved correctly, because they were similar to textbook problems and were familiar from classroom instruction. Three major error patterns were found most frequently as she solved the six problems: 1. selecting parts/all of the numerical data, correctly/incorrectly and yet unrelated to the question; 2. she carried out more than a one-step solution. Both types of errors were found while solving problems #2 with missing elements, and problems #3 and #4 with surplus elements; and, 3. Selecting data needed/related to the question but planning incorrectly was found while she was solving problems #3 and #4 with surplus elements, and problem #6 with missing elements.

Cognitive strategies and task performance among the 4 subjects

The following analysis was based upon the data in Tables 2a (subject 1), 3a (subject 2), 4a (subject 3), 5a (subject 4), and Table 6 (cognitive strategies, task performance, and error patterns among the 4 subjects). The numbers found in the tables did not illustrate anything related to the frequency of behaviors, but specifically indicated the presence of cognitive strategies and task performance of the subjects.

1. According to Tables 2a and 4a, the 4 types of strategies commonly found in subject 1 (male non-M.L.D.) and subject 3 (male M.L.D.) while they were performing task A word problems were:
   a. planning,
   b. rereading the problem,
   c. systematic trial-and-error,
   d. making hypotheses.

2. Subject 1 could indicate all the surplus elements in problems #2 and #4 and all the missing elements found in problem #6.

Subject 3 did not recognize one of the surplus elements in problem #4 but he could indicate all the surplus elements in problem #2 and all the missing elements in problem #3.

Table 6: Cognitive strategies, task performance, and error patterns among the 4 subjects

	Total events			
Subjects	Non-M.L.D.		M.L.D.	
	1	2	3	4
Tasks	A	B	A	B
Strategies				
a.	5	4	3	4
b.	4	1	2	1
c.	3	2		1
d.			1	
e.	1			
f.	1	1	1	2
g.				1
h.	2	1	2	
i.	5		4	1
j.	5		4	1
k.	2		3	
l.	2		3	
m.				
Outcome	6	3	5	4
Error patterns				
A.		3	1	3
B.		1		1
C.	1	1	1	3
D.		3		3
E.	1			

3. Both subjects generated hypotheses while solving problems #3 and #6 with missing elements. The male non-M.L.D. subject made inappropriate hypotheses while solving problem #3, and did not recognize any missing elements. Evidence of this was in his verbal protocols: "Well, it says here 9 of the books and just says some books. I presume that 9 of the books is all the books . . . . So, you add these up and you'll get the answer, 294 dollars."

On the other hand, the male M.L.D. subject generated appropriate hypotheses while solving problem #3 with missing elements; this was illustrated in his verbal protocols: "Well, actually I couldn't find out the answer to that question . . . . So, I've got to find out how many books it had and how much the books cost . . . . Say, there's 5 books and 13 dollars each. Then, I would add . . . multiply 13 times 5 and plus that . . . . I add that to the 9 books and 16 records, and add it to the 7 posters."

However, the male non-M.L.D. subject could indicate the missing elements with appropriate hypotheses. This was illustrated in his oral plan while solving problem #6: "What I need to know is, maybe how many seats are found in each row. May be they can give me there're 7 rows and 42 chairs were altogether. How many seats are found in each row? Well, you maybe divide 7 into 42. Maybe 6 to each row." The male M.L.D. subject could indicate one of the missing

elements (the total number of seats in the music hall) but he made inappropriate hypotheses while trying to figure out the second missing element. The inappropriate hypotheses he indicated in his written scripts were: size of stage, size of building, size of seats, and space between rows.
His verbal protocols also served as explanations of the hypotheses he generated: "Because I would know how many rows are and the size of the building. Oh! I also have to . . . Space and rows or between rows . . . O.K., I have to know the size of the seats. To see how many seats will fit into the building. I have to see the space between the rows or the seats . . . . Say, the space is 500 meters, could be no row. Like, could be any number. Could be just a foot and fit more chairs in. The more space between them, the less chairs you can fit in."

Both subjects generated inappropriate hypotheses while solving problem #3 (male non-M.L.D. subject) and problem #6 (male M.L.D. subject). They also failed to check their hypotheses but decided to follow their original oral plans; this resulted in an incorrect problem resolution.

4. All of the verbalizations presented by subjects 1 and 3 corresponded to their written scripts with one exception: one segment of the verbal protocol presented by subject 3 while solving problem #5 did not correspond to his written scripts. His verbal protocols stated: "I take 15

divided by 735 . . . ." However, his written scripts indicated 735 divided by 15. His verbalizations were thus analyzed as deviating from his written script.

5. According to Tables 3a and 5a, subject 2 (female non-M.L.D.) and subject 4 (female M.L.D.) shared 4 types of strategies in common while performing task B word problems:

a. planning,
b. rereading the problem,
c. restating the problem in own words,
d. making systematic trial-and-error.

6. Neither subject recognized any missing elements in task B, problems #2 and #6. The female non-M.L.D. subject did not recognize any missing elements in problems #2 and #6 or any surplus elements in problems #3 and #4. The female M.L.D., on the other hand, did not recognize one of the surplus elements in problem #4. An example of the female non-M.L.D.'s failure to indicate any missing elements while solving problem #2 was illustrated in her oral plan: "I do subtract adding to that each of the numbers . . . . How much he spent in 2 weeks. O.K., he took out 768 dollars. Then, he spent 359 dollars plus he spent 27 dollars . . . . You add the 9 and 7, 16 . . . 386. Subtract from 768 . . . . He spent 382 dollars . . . . I used what he spent for the bike, the records, and skating shoes. I subtracted it from the amount he took out. Add those two together." She selected all of the numerical data

by following a habitual pattern of arithmetical operation without recognizing any missing elements. She also did not realize the question asked for the amount of money spent but not how much of it was left; this was found in her written answer. Another example of the female M.L.D.'s failure to indicate any missing elements while solving problem #6 was illustrated by her oral plan: ". . . there're 7 days in a week and he stopped doing it for 2 days. So, that would be 5 days left and you times 15 times 5 and that's how many he did this week." She misinterpreted the given information "a few days" as 5 days without recognizing any missing elements, resulting in an incorrect solution.

Evidence of the female non-M.L.D.'s failure to indicate any surplus elements while solving problem #3 was illustrated in her oral plan: "O.K., Kristy has 15 albums. 79 U.S. and she's got 36 Canadian stamps. You add 79 and 36 together. She's got 115 stamps and divide that into. Divide 15 into 115." She actually used all of the surplus elements; this led to an incorrect problem resolution. Evidence of the female M.L.D.'s failure to indicate one of the surplus elements while solving problem #4 was found in her oral plan: "I used the 748 and the 139 for his grocery bill and phone bill. I added 139 and 14 together and I got 153 . . . . It tells me how much he paid for his phone bill and grocery bill. Then, I subtract that from 748 and I got the number that he paid for his housing rent."

An incorrect solution was arrived at, due mainly to her selection of a surplus element, $14, which was unrelated to the question.

7. Verbalizations presented by subject 2 in problem #4 did not correspond to her written scripts.

Verbalizations presented by both female subjects while solving problems #2 and #3 did not correspond to their written scripts. Examples could be found in the female non-M.L.D. subject while solving problem #2. She stated orally: "I used what he spent for the bike, the records, and skating shoes ... I subtracted it from the amount he took out. Add those two together." Her written scripts indicated that she carried out addition before doing subtraction. The actual procedure was different from her oral plan. Another example could be found as the female M.L.D. subject solved problem #3. She stated orally: "I chose 15 stamp albums . . . O.K. I took 13 and 129 and times them and I times 36 times 2 . . . 129 times 17 equals 1903." Various statements in her oral plan mention 15, 17, and 13 stamp albums but the original information provided was 15 stamp albums. She actually wrote down 15, 17, and then 129x17 in her written scripts; this deviated from her oral protocols.

Both female subjects did not use the strategy of checking; this is shown in their verbal protocols and written scripts. The female M.L.D. subject did not use the strategy of making hypotheses, but the female non-M.L.D. subject made an inappropriate hypothesis while solving problem #6. It was shown in her oral plan: "O.K. 12 push-ups each day every week. A few means 3. So 15 times 3 . . . is 45." She hypothesized that a few meant 3 and did not recognize any missing elements in the problem, leading to an incorrect solution.

As a brief summary, 3 types of strategies most commonly found among the subjects were:

a. planning,
b. rereading the problem,
c. systematic trial-and-error.

The 2 non-M.L.D. subjects and the female M.L.D. subject shared the same strategy of restating the problem in his or her own words.

On the other hand, the strategy of making hypotheses was commonly found in the 2 non-M.L.D. subjects and the male M.L.D. subject, but none of them checked their hypotheses. Qualitative differences were indicated in their verbal and written data.

The male non-M.L.D. subject recognized all the surplus elements in task A, problems #2 and #4, as well as the missing elements in problem #6. He did not recognize all of the missing elements in problem #3. The male M.L.D. subject could indicate most of the surplus and missing elements in task A but missed one of the surplus elements in problem #4. He also did not recognize one of the missing elements found in problem #6.

The female non-M.L.D. subject did not recognize any missing or surplus elements in 4 of the task B word problems. The female M.L.D. subject could not recognize any surplus or missing elements in task B, except that she indicated one surplus element in problem #4.

A distinctive contrast among the 4 subjects is that subject 1 (male non-M.L.D.) and subject 3 (male M.L.D.) recognized most of the missing or surplus elements in the problems, but subject 2 (female non-M.L.D.) and subject 4 (female M.L.D.) did not recognize most of the problems' missing or surplus elements.

The 2 ordinary problems in tasks A and B (problems #1 and #5) were accurately solved by all the subjects.

All of the verbalizations presented by the male non-M.L.D. subject corresponded to his written scripts. Verbalizations presented by the female non-M.L.D. corresponded to her written scripts in 3 out of 6 problems; this was the case in 5 out of 6 problems for the male M.L.D. subject and 4 out of 6 problems for the female M.L.D. subject.

Error patterns among the 4 subjects

The following analysis was based upon the data in Tables 2b (subject 1), 3b (subject 2), 4b (subject 3), 5b (subject 4), and Table 6 (cognitive strategies, task performance, and error patterns among the 4 subjects). The analysis followed the 5 specific error patterns, namely:

Error pattern A: selects parts/all of the numerical data, correctly/incorrectly related to the question.

Error pattern B: applies all numerical data and/plans incorrectly.

Error pattern C: selects data needed/related to the question but plans incorrectly.

Error pattern D: carries out more than 1 step for the main solution.

Error pattern E: computational error.

The numbers found in all the tables indicated the presence of cognitive strategies and/or error patterns of the subjects but did not refer to the frequency of behaviors.

As is shown in Table 2b, subject 1 (male non-M.L.D.) made 2 errors in problems #3 and #6 while performing task A:

a. Error pattern C in problem #3 (with missing elements).

b. Error pattern E in problem #6 (with missing elements): computational error in regrouping.

Table 4b indicates that subject 3 (male M.L.D.) made 2 errors in problems #2 and #6 while performing task A:

a. Error pattern A in problem #2 (with surplus elements).

b. Error pattern C in problem #6.

Both subjects 1 and 3 shared error pattern C and a problem with missing elements.

As is illustrated in Table 3b, subject 2 (female non-M.L.D.) made 4 errors in problems #2, #3, #4, and #6 while performing task B:

a. Error patterns A and D in problems #2, #3, and #4.

In addition to error patterns A and D shown in problem #4 (with missing elements), error pattern B was also found in the problem.

b. Error pattern C in problem #6 (with missing elements).

Table 5b indicated that subject 4 (female M.L.D.) made errors in problems #2, #3, #4, and #6 while performing task B:

a. Error patterns A and D in problems #2, #3, and #4.

b. Error pattern B in problem #2 (with surplus elements).

c. Error pattern C in problems #3, #4, and #6.

Both subjects 2 and 4 shared error patterns A and D in common and problems with missing elements (problem #2) and surplus elements (problems #3 and #4).

To summarize: error patterns A, B, C, D, and E were found in the two non-M.L.D. subjects. Error patterns C and E were specifically found in subject 1 (male non-M.L.D. when performing task A) whereas error patterns A, B, C, and D were found in subject 2 (female non-M.L.D. while performing task B).

Error patterns A, B, C, and D were found in the two M.L.D. subjects. Error patterns A and C were found specifically in subject 3 (male M.L.D. performed task A), whereas error patterns A, B, C, and D were found in subject 4 (female M.L.D. performed task B).

The two male subjects shared error pattern C problems with missing elements found in task A.

The two female subjects shared error patterns A, B, C, and D problems with missing or surplus elements found in task B.

Error pattern A was found in subjects 2 (female non-M.L.D. in task B, problems #2, #3, and #4), 3 (male M.L.D. in task A, problem #2), and 4 (female M.L.D. in task B, problems #2, #3, and #4). Error pattern A was also related to problems with surplus elements (problems #3 and #4 in the two female subjects, problem #2 in the male subject) or missing elements (problem #2 in the two female subjects).

Error pattern C was found in all 4 subjects. The same type of problem with missing elements was also found in 3 subjects (problem #6 in the two male subjects and the female non-M.L.D. subject), whereas a problem with surplus elements (problems #3 and #4) was found in the female M.L.D. subject.

To summarize the findings briefly, the 4 subjects executed similar cognitive strategies while performing either task A or task B. Similar error patterns were most distinct in the two female subjects (error patterns A, B, C, and D) while they were performing task B and the two male subjects (error patterns A, C, and E) while they were performing task A. The execution of cognitive strategies might not lead to accurate problem resolution. Evidence that strategies of planning and making hypotheses do not provide a guarantee for accurate problem resolution was indicated earlier in the analysis of cognitive strategies. The latter could be influenced by individual differences in cognitive control of: 1. recognizing surplus or missing elements found in the tasks; 2. rechecking behavior; and/or 3. selective attention relating to analytic-synthetic mental activity and focusing on heeded/unheeded information. This leads to questions about examining individual differences in quality and flexibility while those individuals execute cognitive strategies during task performance.

Discussion

This section will discuss the two research questions posed in Chapter 3, which focused on the executive processes of strategies in relating to the four subjects' task performance.

Research question #1

The first research question asked what and how cognitive strategies would influence the qualitative differences and similarities of task performance found among the average-achieving subjects and the mathematically learning disabled subjects.

It was found that strategies related to planning, rereading the problem, and making systematic trial-and-error were commonly used by all four subjects. The strategy of restating the problem in one's own words was found in the two non-M.L.D. subjects and the female M.L.D. subject, whereas the strategy of making hypotheses was commonly found among the two non-M.L.D. subjects and the male M.L.D. subject. Although the planning strategy was most commonly used by the four subjects, it did not guarantee successful solutions for some of the problems. This was also found in the male non-M.L.D. (average achieving) subject, who did not recognize any missing elements found in task A, problem #3, even though he planned, reread, and restated the problem in his own words. He made an inappropriate hypothesis resulting in an incorrect solution. It would have been impossible to find out how or why incorrect problem resolutions occurred without a qualitative in-depth analysis of verbal protocols, written scripts, and error patterns. These three major evaluating resources indicated how they came to monitor cognitive strategies, which would be related to sensitivity, and to a deliberate effort to control flexibly while attending to task performance.

With reference to the previous discussion on the cognitive performance between L.D.s and non-L.D.s, presented in Chapter 2, Brown and Smiley's (1978) explanations of the disabled learners who have insufficient knowledge or less practice with how and when to regulate cognitive strategies, Torgesen and Silver's (1979) suggestion of learning disabled individuals who do not utilize the same cognitive strategies in task performance as the normals, and Wong's (1982) comment of the learning disabled using a "primitive" organized strategy provide some thoughts for the following discussion. In addition, Borkowski (1985), Cawley (1985), Blankenship (1985), and Sternberg's (1985) notions on strategy transfer in relating to higher-order executive processing, proficiency, and consistency expressed in various tasks and performances, and in novel situations can be incorporated with current findings of this study.

The 2 M.L.D. subjects utilized similar cognitive strategies to those utilized by the 2 non-M.L.D. subjects during their task performance. Differences were found in their styles of orchestrating the strategies and their efforts to attend to selective information required for problem resolution. Reflective thinking behaviors while performing the task were most commonly found in subject 1 (male non-M.L.D.) and subject 3 (male M.L.D.) while performing the task. The two female subjects are quite different from the two male subjects in terms of indicating their reflective thinking behaviors while planning for the solution steps. Examples of reflective thinking behaviors in subjects 1 and 3 can be found when they were able to relate the questions to the information given in the problems. This required them to analyze and synthesize various elements needed or not needed for problem resolution, particularly for the problems with surplus or missing elements. On the other hand, subject 2 (female non-M.L.D.) did not recognize any missing or surplus elements presented in the four specific problems. Her error patterns were found to be similar to those of subject 4 (female M.L.D.), who recognized only one surplus element in one of the problems. It appears that reflective thinking behavior found among the four subjects is related to individual differences in cognitive styles, corresponding to their control of focus upon heeded or unheeded information, which influences their intentional behaviors to monitor the strategies. Gerber (1983) gives two examples of behavioral tendencies found among reflective learners. These learners tended to scan the visual stimulus in an orderly and obviously deliberate manner to detect categorical elements or discriminative features. This was interpreted as evidence of a specific cognitive search strategy. Another tendency was related to delaying responses while thinking or evaluating solution alternatives. Both kinds of tendencies were apparently intentional behavior. Gerber also argues that strategies represented the operation of cognitive control processes that influenced the depth of processing required to reduce response uncertainty. Ulibarri (1982) found that processing strategy could also be influenced by an individual's cognitive style with respect to field dependence/independence. He explains that the field-dependent learners were more likely to be overly sensitive to misleading gestalt-like cues than fail to utilize maximum processing capacity. This could be the result of unfamiliarity with the elements of the task or differences in experience. He stresses that cognitive style tends to be a function of natural experience and familiarity with the task situation.

The poor performance found in the two female subjects was mainly related to their selection of part/all of the numerical data, related or unrelated to the questions (error pattern A), selecting data needed/related to the question but planning incorrectly (error pattern C), and carrying out more than one step for the main solution (error pattern D). Evidence was found that the two female subjects shared error patterns A and D in common while both solved task B problems #2 (with missing elements) and #4 (with surplus elements). They also shared error pattern C in common while both solved task B problem #6 (with missing elements). Their errors were also related to their "number crunching routine" approach to the problems. Examples could be found in the female non-M.L.D. subject who selected all of the numerical data unrelated to the question, while solving problem #2 with missing elements. A habitual pattern of using a "number crunching routine" was evidenced. The female M.L.D. subject indicated her "number crunching routine" pattern while solving problem #2 (with missing elements) by means of applying all numerical data as well as striving for all possible operations in her two different plans. She did not recognize any missing elements, which resulted in an incorrect solution. Their major difficulty could be explained by Mikhal'skii's (1975) comment that if learners do not dwell long enough on the conscious mastery of the conditions in solving mathematics problems, they become adapted to a habitual/mechanical rote-computational pattern for problem resolution without analyzing or understanding the conditions and questions presented.

Similar notions are shared by Wien (1983), Gerber (1983), and Brown and Smiley (1982). Gerber (1983) further comments on the poor performance found in the learning disabled:

> The significance of the dysfunction, I believe, lies in the need to detect "blind rule-following" which makes the child, neither optimally efficienct on a given task nor more adaptive with respect to the range of tasks he or she will encounter. (p. 259)

Another supportive argument can be traced by referring to the influential power on cognitive development of cumulative learning and past experience, as mentioned in Chapter 2. If basic conceptual knowledge, such as mathematical facts, is interpreted as a standard rule, then cognitive strategies may serve as mechanisms of rule-governed behavior. Each of the four subjects has his or her unique repertoire of conceptual knowledge, as indicated by their results found in the Canadian Achievement Test and their current classroom performance. Qualitative differences among individual subjects were found while they were executing cognitive strategies during task performance. This was particularly noticeable when they were solving the two categories of non-textbook mathematics word problems. Their poor performance may be related to insufficient practice or less knowledge of goal-directed cognitive strategies incorporated with conceptual knowledge. Steffe, Richards, and Von Glasserfeld (1979) suggested that:

> concepts, structures, skills, or anything that is considered knowledge cannot be conveyed ready-made from teacher to student or from sender to receiver. They have to be built up, piece by piece, out of elements which must be available to the subject. (p. 43)

Piper and Deshler (1985) supported a similar method and commented that:

> because of the rapid fire pace to finish the textbook by the end of the year or a given grading period, limited opportunities for mastery learning are afforded the LD student . . . . Clearly, the presentation of a skill within a curriculum carries with it no assurance of mastery of the skill. The implications are obvious; everyone is expected to learn the same amount of information in the same time period . . . . As the LD student progresses through the secondary grades the gap between his/her skills and the demands of a fixed curriculum become increasingly greater. (p. 36)

The poor performance of the two female subjects can also be related to selective attention, which influences how an individual focuses or shifts from focus on relevant stimuli. Treisman (1969) also suggested that appropriate strategies are required in the processes of selective attention. Krupski and Bryan (1981) proposed that there was an interaction among particular child, task, and setting characteristics which influenced an individual's attention. Their idea is that voluntary attention has a strong volitional component that requires conscious monitoring and effort. Examples related to attention were found among three subjects in this study. Verbalizations presented by subject 3 (male M.L.D.) while solving task A, problem #5 did not correspond to his written scripts, as mentioned in the data analysis earlier. He did not deliberately attend to what he was saying which did not correspond with what he was actually writing. Verbalizations presented by subject 4 (female M.L.D.) while solving task B, problem #2 did not correspond to her written scripts. This was indicated when she verbalized differently the number of stamp albums given in the problem. This also was mentioned in the data analysis. A similar instance was found while subject 2 (female non-M.L.D.) was solving the same problem. She verbalized that the question asked the number of stamps in all the albums, but her written answer and verbal statements indicated an incorrect solution given for the number of stamps in each album.

On the other hand, the two M.L.D. subjects were not considered as disorganized during their task performance. Planning strategy applied along with their verbal protocols and written scripts provide some evidence of this. As well, subject 3 (male M.L.D.) utilized the strategy of drawing a diagram while solving task A, problem #6. He was the only subject who utilized such a strategy in this study. It can be argued that individual differences are found in the M.L.D. subjects. Subject 3 (male M.L.D.) did not solve the specific problem with missing elements entirely successfully, although he utilized the strategy of drawing a diagram. His failure was mainly due to setting up inappropriate hypotheses. He was indeed well-organized in setting up his plan while looking for the missing elements.

Moreover, neither mediational deficiency (as related to structural deficiency due to inability to employ strategies) nor production deficiency (as related to strategic deficiency due to failure to use strategies) was found in the mathematically learning disabled subjects. If it is necessary to redefine strategy deficiency based upon its original accent, it can be suggested that the two female subjects involved in this study are operating at a lower level of proficiency and consistency while executing strategies inflexibly for novel task demands. A brief review of related data will proceed before providing supportive arguments already mentioned in Chapter 2. Evidence indicated that some specific commonalities in strategies employed and error patterns made were found in both types of subjects, as illustrated previously. It was mentioned earlier that reflective thinking behaviors, as expressed by intentionality to monitor task-appropriate strategies during the process of cognitive search, were most commonly found in both types of male subjects. On the other hand, both types of female subjects seldom executed analytic-synthetic mental processes in order to relate the questions to the problems but were used to a habitual pattern of "number crunching routine" to approach the task. Their cognitive control processes in relating to the depth of processing required to reduce response uncertainty and the regulation of task-appropriate strategies are less flexible and proficient than the two male subjects during task performance. The four subjects could solve all the ordinary problems accurately with spontaneous mathematical/computational tactics. Difficulties were encountered while solving the two specific categories of problems with surplus or missing elements. Supplementary information of the subjects' current mathematics standard was also gained from their pre-assessment of mathematics concepts and the application test battery found in the Canadian Achievement Tests. There are two problems which ask for missing information in a more obvious way with multiple choices provided. The 2 problems are:

1. A jet travelled from Toronto to Vacouver in 5 hours. What more do you need to know to find the average speed of the jet? (5 multiple choices were given.)

2. What is the missing number?
   27, ____, 3, 1

Computerized results from the subjects indicated that subjects 1 (male non-M.L.D.) and 3 (male M.L.D.) could solve both problems correctly but subject 2 (female non-M.L.D.) missed the second problem and subject 4 (female M.L.D.) missed both problems.

Sternberg's (1985) explanations of distinctive differences between automatic and controlled processes, presented in Chapter 2, suggest that an individual depends on automatic, local processing when processing information from a locally applicable knowledge base in which considerable expertise is already acquired. On the other hand, if an individual does not have much expertise while processing information from new domains, then controlled, global processing is necessary. The controlled processing, functioning at higher-order, activates the knowledge-acquisition components, functioning at lower-order, to handle the new situations. The four subjects involved in this study could easily and correctly solve the ordinary problems which depended primarily on automatic processing based upon previous knowledge acquired. Besides, the ordinary problems were familiar from classroom instruction. Declarative knowledge (knowing the task characteristics as well as personal abilities) and procedural knowledge (knowing how to execute various actions or procedures) are not sufficient if an individual does not know when and why to monitor strategic behaviors intentionally to achieve various tasks demands and goals. This was indicated in the subjects' task performance, specifically with the novel problems with surplus or missing elements. Evidence of this was particularly noticeable in subject 2 (female non-mathematically learning disabled) and subject 4 (female mathematically learning disabled), who frequently applied the "number crunching routine" or the "blind rule-following" method to solve the three types of word problems without pondering how and when to modify old knowledge for new situations. The two subjects utilized both declarative and procedural knowledge quite efficiently in solving the ordinary problems, but were not proficient enough to know when to execute conditional knowledge for solving problems with surplus or missing elements. They were more inclined to present rigid and inconsistent responses rather than take time to think and adjust task-appropriate strategies deliberately. At the same time, Case's (1985) suggestion of strategy control requires a set of global cognitive and affective processes which are domain- and expertise-related. It coincides with Cawley's (1985) and Blankenship's (1985) ideas of proficiency and consistency of accurate response in various time and situations. Both female subjects' failure to detect most of the surplus and missing elements embedded in the problems appeared to be mainly due to their insufficient experience in conditional knowledge as related to both proficiency and consistency in executing task-appropriate strategies while confronting novel problems.

The two male subjects demonstrated higher proficiency and consistency in monitoring task-appropriate strategies. Their reflective thinking process was utilized in analyzing and synthesizing information, and was qualitatively different from the two female subjects.

If the two male subjects were considered as experts and the two female subjects were considered as novices, then the fact could best be explained by Sternberg's (1985) idea that:

> Experts are at an advantage in their domain of expertise, because their ability to stay for longer amounts of time in the better-developed local processing subsystem enables them to free global processing resources for what, to them, are new situations. Novices are overwhelmed with new information, and must engage global resources so frequently that most of the new information that is encountered is quickly lost. Experts are thus better able to handle familiar tasks within the domain of expertise, and also to learn new tasks, since global processing resources are more freely available for the intricacies of the situation confronted.

The two male subjects' high proficiency and consistency in their task performance could also be related to Borkowski's (1985) suggestion of strategy transfer upon making a decision to modify previously learned knowledge and strategies for present tasks demands. The same issue of strategy transfer could explain the two female subjects' lower proficiency and flexibility in executing appropriate strategies for novel task demands. Gerber (1983) and Swanson (1985) supported the notion that the learning disabled were not strategically deficient but they were inflexible in applying more efficient strategies.

It can be concluded that differences in flexibility to regulate the cognitive strategies produced qualitative differences among the four subjects although similar strategies were utilized.

Research question #2

The second research question asked if cognitive strategies would be applicable across the task performance on four different arithmetical operations and the three major categories of mathematics problems.

The findings indicated that some cognitive strategies were found to be in common among the four subjects, and between two given subjects, such as the female normal and female M.L.D. subjects, in performing the two tasks. At the same time, rereading the problem; restating the problem in their own words; making systematic trial-and-error; making hypotheses; and planning strategies were most often found among the four subjects. Although the two tasks involved three similar categories of mathematics problems and two different arithmetical operations in each task, similar cognitive strategies were utilized by the subjects. Thus, cognitive strategies are applicable across the two tasks.

However, the application of similar cognitive strategies does not guarantee successful solution of the problems. This mainly depends upon the subjects' flexibility in regulating their strategies while choosing or integrating information required for problem resolution. Hayes-Roth's (1977) concept of cognitive change and Senf's (1971) notion of information integration provide supportive arguments. Hayes-Roth (1977) suggested that cognitive change, influenced by the "to-be-learned" information, occurred as learning progressed. Senf (1971) explained that an individual acts on information by assimilating it into his/her past cognitive-experiential repertoire. An example can be found in the female M.L.D. subject's task performance on task B word problem #2. She changed her plan for problem resolution, but her focus was on the application of arithmetical operation and numbers given. She did not recognize any missing elements; nor could she interpret the question correctly. She was not considered as entirely inflexible, though her blind rule-following and number crunching routine approaches to the problem provided evidence of failure strategies. Liedtke (1979) explained that:

> some slow learners are excellent, indeed, at remembering all the rules they have ever heard. Occasionally a young child will want to recite every known rule and then apply these rules to the tasks on hand in what amounts to a random fashion. (p. 14)

These findings also support Gerber's (1983) comments that tactics are clusters of skills or procedures associated with solving specific types of problems, such as applying the four basic arithmetical operations for solving the ordinary problems found in this study. He explained that strategy referred to skillful, deliberate, and coordinated use of problem-solving tactics. This agrees with Lawson's (1984) suggestion that strategies will facilitate task performance, but there is no guarantee of successful solution of the task. In other words, a strategy's effectiveness depends upon how flexibly it is applied and regulated.

At the same time, productive thinking, in terms of regulating cognitive strategies while confronting a novel task demand, is actually required for problem resolution. Productive thinking as a basic channel that leads to cognitive search was discussed in the last section of Chapter 2. It is also suggested by Covington, Crutchfield, Davies and Olton (1972) that:

> productive thinking is the use of the mind in an effective, intelligent, and creative way directed toward to solution of a problem. It is reflected in every type of human problem solving . . . . It requires that a large number of different thinking skills be brought into play. Which of these skills will be particularly relevant in a given situation depends on the specific nature of the problem at hand. Productive thinking . . . . may involve the skills of identifying the essentials of the problem, generating outcomes of the experiment . . . . The crucial point is that all these different thinking skills enter in some degree into all productive thinking activities. (p. 2)

As a brief summary, the qualitative differences and similarities found among the four subjects are indicated by the strategies utilized in corresponding to their error patterns. A strategy of checking/looking back was not found among all the subjects. Individual differences in approaching the word problems and their unique ways to regulate cognitive strategies while analyzing information related to the questions influenced qualities of task performance. Strategies themselves do not guarantee successful problem resolution; success depends on how flexibly and proficiently strategies are monitored while solving novel problems in various situations.

CHAPTER 6

CONCLUSIONS AND IMPLICATIONS

Conclusions

This study intended to systematically assess the execution of cognitive strategies found among average achieving and mathematically learning disabled individuals. The major findings of the present research indicated that an individual's flexibility when monitoring task-appropriate strategies influences the quality of task performance. At the same time, the results indicated that cognitive strategies are applicable across the task performance on four basic arithmetical operations and the three specific categories of mathematics word problems designed for the study.

In light of the research findings, support is provided for the notion that an individual's achievement of task performance involves much more than acquiring a collection of content-oriented tactics, such as the computational skills required in performing some mathematical tasks. It is indeed heavily dependent on how one regulates one's own strategic thinking while searching for the direction of problem resolution. Polya (1970) commented that the mastery of mathematics did not refer to

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an individual's ability to solve standard problems, such as the ordinary problems designed for this study, but that mastery was the ability to solve those problems requiring a known independence of thought, common sense, and inventiveness. Referring to the focus of this study, Polya's comments on independent thought are interpreted as strategic thinking and decision making, common sense is explained as the resources of strategies available, and inventiveness required is how flexibly a learner conducts a cognitive search for either self-generated or learned strategy, be it well-established or to-be-constructed. In this study, the four subjects had their unique repertoires of conceptual knowledge. However, individual differences in quality and flexibility of strategy execution and monitoring were particularly obvious while they were solving the two categories of non-textbook mathematics word problems found in both tasks. Although the male non-M.L.D. subject could not indicate one of the missing elements in task A, problem #3, and the male M.L.D. subject did not notice a surplus element (problem #2) and a missing element (problem #6) while performing task A, they both could relate most of the questions to the required information given in the problems. In addition, the male M.L.D. subject was the only subject in this study who utilized the strategy of drawing a diagram.

Be it a self-generated or a learned strategy, he could apply it flexibly and spontaneously while solving one of the problems with missing elements (problem #6), but he made inappropriate hypotheses which led to an incorrect solution. On the other hand, the two female subjects did not recognize any missing or surplus elements in the four specific problems found in task B, with the exception that the female M.L.D. subject could indicate one of the surplus elements found in task B problem #4. Their poor performance was related to a number of facts concerned with insufficient practice or less experience at modifying goal-directed cognitive strategies incorporated with conceptual knowledge, insufficient selective attention while focusing and choosing information required for problem resolution, and insufficient reflective thinking behaviors. Differences found in the reflective thinking behaviors among the four subjects would be related mainly to their individual differences in cognitive styles. These influenced their intentional behavior to monitor the cognitive strategies during task performance.

Moreover, the research findings did not support the commonly accepted notion of strategic deficiency and inactive behavior in task performance. There was no evidence of strategic deficiency, as related to failure to use strategies, nor inactive behavior found in the two mathematically learning disabled subjects who participated in this study. However, an expanded idea of strategy deficiency could specify that the two female subjects were not proficient nor consistent enough to execute task-appropriate strategies flexibly. The female non-M.L.D. subject performed the task in an inferior manner when compared with the male M.L.D. subject. This was due mainly to the fact that she followed a habitual pattern of "number crunching routine" for problem resolution without dwelling in deeper thought. These characteristics were marked by evidence that her verbal protocols and error patterns were concerned mainly with applying all numerical data, planning incorrectly, and striving for all possible arithmetical operations. Both female subjects did not initiate initial metacognitive acts of pondering expressed by deliberate intention to think deeply before executing goal-directed strategies for problem-resolution. Being able to follow the rules, such as the basic mathematical facts, is no guarantee for accurate problem resolution, which is related directly to rule-governed behavior. Scandura (1970) explained that:

> an operational definition of what is learned must be formulated relative to a given class of rule-governed behaviors. Any such definition must be based on performance on a small, finite number of instances, and if possible, should be applicable no matter how many test instances are employed. (p. 528)

Furthermore, Carpenter, Corbitt, Kepner, Lindquist, and Reys (1980) supported that position:

> problem solving is the process of applying

> previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with nontextbook problems . . . . In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. (p. 216)

Switching to the topic of inactive behavior during task performance, the present findings, particularly from the data of verbal protocols and written scripts, did not provide any evidence of inactive behavior among the subjects. Although Keogh (1973) has cautioned that cognitive styles only present a specific pattern and organization of cognitive controls, the possibility still exists that if a learner experiences continuous failure, that fulfills low achievement expectations, and thereby further confirms low academic self-concepts and inactive learning behavior. The findings indicated that the two male subjects were able to execute analytic-synthetic thinking behavior during task performance more effectively than the two female subjects. They were more sensitive to their intentional and goal-directed efforts in deploying strategies for problems resolution.

It can be argued that knowing some strategies does not guarantee knowing how and when to deploy them flexibly and appropriately. Gerber (1983) presented supportive comments that the learning disabled were viewed as strategy inefficient or strategy inflexible. Would it be more desirable to consider seriously Brown's (1978) notion of recontextualizing school experiences by means of teaching the learning disabled some learning strategies?

Implications

Educational implications

The qualitative differences and similarities found in the four subjects during their tasks performances were influenced by a number of factors concerned with their levels of proficiency in monitoring goal-directed cognitive strategies corresponding to: the task demand; the individual's repertoire of conceptual knowledge; selective attention; and reflective thinking behaviors. Suggestions can be made on teaching learning and thinking strategies. Several researchers advocate the teaching of metacognitive skills that will help learners, especially students with learning problems, to use appropriate learning strategies (Flavell, 1981; Keogh, 1977; Wiens, 1983). Flavell (1981) explained the importance of teaching students to become aware of and to maintain an awareness of their learning and communicating goals. This involves the students in choosing means to a goal, and assesses the effectiveness of those means while monitoring their performance. Flavell also suggested teaching them how to evaluate detected problems in learning and communicating. This provides an efficient way to help students overcome their processing problems, if they have any. Brown (1978) also claims the importance of recontextualizing school experiences and providing meaningful educational activities for the learning disabled by means of teaching them metacognitive skills.

Deshler, Schumaker, and Lenz (1984) introduced a comprehensive intervention model for learning disabled adolescents. Their model encompasses five components stressing motivation, skill acquisition, generalization, curriculum, and communication. This section will highlight some of the critical notions. The motivation component plays an important role in their intervention model since the learners have to be involved fully in both the skill acquisition and generalization phases. If learning disabled individuals are inactive or lacking intrinsic motivation to learn, this will pose a serious obstacle to teachers in delivering new skills to them or in expecting any cognitive change. Brown (1978) comments that a learner's attempt to control his/her academic achievement would be vitiated if he/she did not believe in himself/herself as an active agent in knowing what there was to know at school and had no expectations of his or her ability to control task performances. Meichenbaum (1977) supported the notion that cognitive changes are closely related to motivational and behavioral processes. He does not separate affective and cognitive components, since cognitive change should not be seen with a purely "intellectual" insight. Goal-setting is also found to be important in the motivation component. Gold (1978) suggested that:

> goal-directed thought involves the activation of an executive scheme which is a plan for solving particular problems encountered. (p. 11)

Meichenbaum (1977) believed that when an individual confronted a task, setting up challenging goals would make the task more meaningful. This would facilitate task performance by helping to maintain a high level of attention.

The model also follows Meichenbaum's (1975) cognitive-behavioral approach for skill-training, such as self-monitoring, self-evaluation, and self-reinforcement. It is believed that self-monitoring allows an individual to attend deliberately and carefully to his/her own behavior facilitated by self-verbalization of actions. Self-evaluation/correction is a discrimination response, a matching which reveals the discrepancy between what one is doing and what one ought to be doing. A close match between performance criteria and information from feedback should result in some satisfaction with oneself, while a large discrepancy will yield dissatisfaction. Self-reinforcement is generated by the learners who are able to continue error correction themselves (Deshler, Schumaker & Lenz, 1984; Meichenbaum, 1975). Dennis, Foster, and Maxwell (1981) taught goal-setting, self-recording, and self-reinforcement to some learning disabled junior-high school students.

The results indicated that the self-instructional approach was time-consuming and it did not determine the effects of training on actual academic behaviors of the students. In another study, Chatman, Johnsen, Tollefson, and Trach (1983) taught learning disabled junior high students to set realistic goals, develop plans, self-monitor and evaluate their performance, and accept responsibility for the outcomes. Their results showed increased rates of assignment completion in the resource room and in the regular classroom. The self-control methods were in effect. The differences between the results of these two studies may be related to how the self-control methods are delivered by the instructors. On the other hand, Gerber (1983), Hallahan and Sapona (1983) questioned the usefulness of self-monitoring techniques, such as cognitive behavior modification in academic remediation. Hallahan and Sapona (1983) commented that:

> It is our opinion that the self-monitoring techniques work when children are working on tasks for which they already have the skills. In other words, we are skeptical about how successful the procedure would be for children when they are in the acquisition stage of learning. (p. 619)

Cawley (1985) supported the idea that self-monitoring should be implemented during overlearning to stimulate rehearsal and for practice.

Wiens (1983) suggested recontextualizing school curricula by means of teaching the learning disabled a system of learning strategies. It seems quite likely that individual learners need to have flexible thoughts before meaningful mathematical instruction begins. This allows them to relate the given information to their experiences and knowledge that already exist and can be retrieved from memory.

The essence of mathematics is the abstraction of the relational content from which new insight will be developed. It is important for individual learners to experience the process aspects of mathematics which do not emphasize performing classroom assignments or solving problems aimed at improving some known arithmetical skills or tactics. Pieper and Deshler (1985) comment on the "chalk and talk approach" to formal mathematics instruction. This approach does not emphasize individual differences in learning rate and style largely due to time constraints and the size of the groups. They suggested that:

> instruction should, therefore, rigorously apply skills to real life applications, problem solving situations and word problems. The application of newly learned math skills to real life situations is critical. This instructional practice necessitates the understanding of concepts and then application to real problem solving activities . . . . the LD student has been found not able to effectively generate strategies in math when left to his own resources . . . . Through the technique of repeated associations and the experience of reconstructing meaningful and familiar examples of the new concept, the new concept itself becomes learned. (p. 40-41)

Swanson and Cooney (1985) suggested that the learning disabled might learn many prescribed rules but fail to transform simple strategies into more efficient forms. Their idea can be expanded to suggest that rote information is best applied in a meaningful or simulated real life situation at the level of strategy transfer. This can be done by means of giving the learners as many opportunities as possible to recognize related skills as a part of a meaningful whole which can serve them in real life. It is definitely needed to ensure that they have the sufficient pre-requisite knowledge required, be it declarative or procedural knowledge, to further develop strategy transfer. Two questions can be generated: 1. Can these strategies, be they learned or self-generated, abide with the learners if they are not regularly exercised? 2. How can the learners be facilitated to achieve far-generalization by means of taking these strategies with them into their real life situations?

On the other hand, the findings of this study illustrated that the two specific poor performers, subjects 2 and 4 (female average-achiever and female mathematically learning disabled), adopted a habitual pattern of "number crunching routine" to approach the mathematics word problems. Mechanical decision of arithmetical operations and product-oriented procedures of solution were most commonly found in these two subjects but analytic-synthetic mental processes and sustain attention were very seldomly found in these two subjects. At the same time, checking behavior, in terms of re-evaluating the results and operational plans or procedures, was not found in all four subjects. This may be due to the fact that calculators were not commonly allowed to be used as a learning device in the classroom setting. The limitation of time allowed for completing classroom assignments often makes the students experience anxiety for task completion. It also adversely influences the learners to sustain attention on the information required for task performance and to develop a high-level thinking ability.

In supporting Wiens' (1983) notion of recontextualizing school curricula, the utilization of calculators in a classroom setting may facilitate the procedures of problem solving. The findings of studies done at the Shell Centre for Mathematical Education at Nottingham University in Britain suggest that calculators reinforce the memory of basic facts, reveal new number relationships and present a means of checking the results of practice calculations and realizing errors in a non-threatening way (Bell, 1981). At the same time, there is much literature which advocates the efficacy of applying the micro-computer as a learning device, particularly for the learning disabled (Goodman, 1983; Grimes, 1981; Messinger, 1983). It has been suggested that:

> problem solving is a skill that the learning disabled need, and strategies related to problem solving can be adapted easily for the computer . . . . The ability of the computer to hide the answers from the student has actually made a decision that prevents second guessing the system, as one might if the answers are fixed or retrievable from a fixed location such as a textbook or workbook. (Schiffman, Tobin & Buchanan, 1982, p. 558)

Torgesen (1983) explained that micro-computers in the classroom:

> can present practice activities in a variety of different formats that can help to maintain interest. They also have an enormous capacity for sustained attention—they can deliver as much closely monitored practice as each student needs. Finally, they have the capacity to both measure the time for responses . . . . which emphasize both accuracy and speed could constitute unique contribution of computer technology to learning disabled children. (p. 235)

As well, Schiffman, Tobin, and Buchanan (1982) reported the observational results of a micro-computer training program for learning disabled students done at the Johns Hopkins University's Training Centre during the summers of 1981 and 1982. These results provide evidence that supports Vygotsky's (1978) notion of zone of proximal development. It was reported that:

> with a minimum of instruction, the children were able to write simple computer programs, indicating the presence of a previously untapped high-level thinking ability and motivation. (Schiffman, Tobin & Buchanan, 1982, p. 558)

Lester (1981) indicates that:

> basic world knowledge typically is background, experience-based knowledge which must be used in order to make sense out of the problem. Planning knowledge enables the subject to "get a feel" for what needs to be done. It helps the individual understand what sequence of actions might lead to a solution. Strategic knowledge governs how basic world knowledge and planning knowledge are to be used in synthesizing a structural model of the meaning of the problem. Strategic knowledge, then, is metaknowledge that the individual uses to direct the processes of making sense out of a problem and deciding what actions to take to get a solution. (p. 62)

Bishop (1981) and Monakhov (1981) introduced both cognitive and affective aspects of mathematical learning. The cognitive aspect of mathematical learning concerns an individual's thinking behavior, whereas the affective aspect emphasizes the mutual communication between the learner and the instructor expressed in a dialectical manner. It is advocated that similar notions can be applied to learning and teaching cognitive strategies. Motivation and personality are also acknowledged as attributes of quality of task performance but they are beyond the scope of this study. However, emphasis on both cognitive and affective aspects will be suggested in the following section for the purpose of developing an educational model of intervention.

Toward a cognitive-dialectical model of intervention

Learning disabled students are usually exposed to remedial strategies that emphasize working with their weaknesses while ignoring their strengths. It is believed to be more promising for researchers and practitioners to devote their efforts to exploring by what means these learners' strengths can be developed more fully. The following discussion will present some related concerned and will introduce a cognitive-dialectical intervention model with reference to the similar notion on mediated learning experience shared by Feuerstein, Meichenbaum, and Vygotsky.

Feuerstein (1979), Meichenbaum (1975), and Vygotsky (1978) addressed the importance of the presence of a more capable mediator during the learning process. These scholars also emphasize the importance of learning and applying specific rules or strategies. The most distinctive similarity found among them is dialectical interaction/communication between the learner and the instructor who assists the learner to explore, try on, and consolidate the conceptualization of the presented problem for task performance in order to produce new and more adaptive behaviors and skills. Meichenbaum (1975) explained this approach clearly in his suggestions for treatment of academic problems. Instructors translate cognitive strategies into sets of self-statements that can be modeled and rehearsed by the learners. Instructors can model not only task-relevant problem-solving self-statements so that the learners know how to cope with frustrations and failure while doing a particular task, but Feuerstein (1982) stressed that the mediation for competence is one of the criteria for a mediated learning experience. He explains that mediated learning experiences are those aspects in which an adult selects and frames the external world in order to focus a learner's attention on those aspects that are most important. Vygotsky (1978) further suggested that higher cognitive functions develop when learners interact with other people in their environment. During interactions, more capable people serve as mediators who introduce strategies for problem-solving.

Feuerstein and Vygotsky share a similar model of learning through interpersonal mental process in which the learner's play active roles in their environments. Vygotsky (1978) suggested that mediators focus the learner's attention on relevant dimensions of the environment and provide the tools, such as speech and memory strategies, for problem solving. Feuerstein (1982) supported the idea that the mediator relates the new experience to the previous event and to those that will occur in the future. The essence of Feuerstein's mediated learning experience is based mainly upon the interpersonal mental process. He explained that although the learners become active explorers in the course of their quest for knowledge of themselves and the external world, it is the mediator who shares his/her intention implicitly or explicitly and helps to initiate some specific activities or to communicate some thoughts. His conceptual model does not emphasize, as Vygotsky's does, the importance of intrapersonal mental processes, which allow an individual to function more independently on his/her own. Vygotsky (1978) argued that all psychological processes are initially social-shared between individuals—but the basic interpersonal nature of thought was gradually transformed through experience into an intrapersonal process. He views the mediator as holding a regulatory and learning function that leads learners to proceed with the mechanism of internalization, from other-regulation to self-regulation. He described internalization as the internal reconstruction of an external operation. He suggested that internalization, by means of social interaction, allowed the generalization of learned skills from interpersonal experience, found between individuals, to intrapersonal experience, operating within an individual. This applies equally well to the idea of to-be-constructed and well-established strategies as advocated by Kirby (1984) and Lawson (1984). Mediated learning experience occurs at the initial stage of the construction of cognitive strategies through interpersonal communication, or it can be self-generated from observation or modeling. The self-regulated function of cognitive strategies operates spontaneously or automatically within an individual once the strategies are well established. This will result in intrapersonal experience of strategies monitoring in correspondence to task performance.

Modeling and verbal mediation are essentially required before strategies can be well-established; they are also being executed automatically and appropriately for various tasks' demands. Deshler, Alley, Warner, and Schumaker (1980) also support the notion that modeling allows learners to witness all of the processes involved in various strategies. Meichenbaum's (1975) self-instructional methods explain that observers gain information from a model and convert it into covert perceptual-cognitive images and covert mediating rehearsal responses, which are retained by the observers and later used as symbolic cues to overt behaviors. His self-instructional methods include explicit modeling of such mediating responses in the form of self-statements that facilitate change in the learning process and behavioral change. On the other hand, Vygotsky's (1978) thesis also emphasizes the relationship between semantic and behavioral components. He explained that:

> speech and action are part of one and the same complex psychological function, directed toward the solution of the problem at hand ... Children solve practical tasks with the help of their speech, as well as their eyes and hands. (p. 25-26)

He continues to explain that learners can use words devising a specific plan, searching for and preparing the means that will be useful in problem solving, and planning for future actions.

Feuerstein's, Meichenbaum's, and Vygotsky's notions of learning potential and mediated learning experience ought to be taken seriously if special educators accept Vygotsky's (1978) belief that:

> Learning awakens a variety of developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with peers. Once these processes are internalized, they become part of the child's independent developmental achievement. (p. 90)

At the same time, Keogh (1977) and Gerber (1983) have suggested that the learning disabled may not be strategy deficient since they use a variety of responses at their disposal. They may prefer or overlearn certain strategies which are applied inflexibly. Therefore, it is more meaningful for special educators to try to facilitate the repertoire of these learners' strategies, and to teach them how to regulate and deploy task appropriate strategies flexibly.

Implications for future research

The descriptive case study approach, analysis of verbal protocols, and error patterns presented in this study suggest some alternatives beyond traditional experimental designs and techniques. Qualitative data analysis based on an idiographic method allows in-depth studies of individual cases. Covert processes of strategy monitoring cannot be identified clearly by quantitative and experimental techniques exclusively. At the same time, some cognitive studies analyze verbal protocols quantitatively, focusing on either correct or incorrect responses, or the frequency of certain behaviors. These provide very little insight as to how an individual performs a task. Homans (1962) suggested that there were neither good nor bad methods, but only methods that were more or less effective in reaching the goals. A better question is to ask if the research method matches the questions asked. The suggestion can be extended to employ a blend of both qualitative and quantitative methods of inquiry. A qualitative nonexperimental design, using verbal and written data to describe the cognitive phenomena systematically observed and recorded in natural settings, will provide a wealth of information which emphasizes ecological validity. Some precautions in collecting verbal data are generated from this study, such as: clarifying the instruction to verbalize by introducing probing questions but minimizing the possibility of interrupting the subjects' thoughts while they are thinking aloud on their task performance; and further probing some specific questions, if required, in order to examine how spontaneous responses which may not be verbalized by the subjects themselves are produced automatically. Although the durations of the exact time spent for task performance are not available due to the introduction of probing questions during task performance, and the generalizability of the verbal protocols is limited, the consistency of the verbal protocols with empirical data, such as the written scripts, as well as the design of objective methods of encoding and evaluating them, can overcome the drawbacks.

To summarize very briefly, mediated learning experience plays a crucial role in facilitating the transition of to-be-constructed strategies, through interpersonal communication, to become well-established strategies regulated within the individuals. Following Vygotsky's (1978) concept of the zone of proximal development, it is believed that cognitive strategies help evoke the learners' actual developmental levels.

Academic achievement requires that students know how to learn, not just what to learn. It has been noted that not all learning disabled students are able to transfer the skills learned from the resource room to the regular classroom while approaching academic tasks (Schumaker, Deshler, Alley & Warner, 1983). Sheinker, Sheinker, and Stevens (1984) presented some cautions for utilizing the cognitive strategies training approach:

> the cognitive strategies approach is not a substitute for direct instruction, which has been demonstrated to be effective in teaching basic reading and math skills . . . . Cognitive strategies should be a distinct, well organized, integral part of the instructional sequence . . . . Mastery of basic skills is prerequisite to acquisition and use of generalized cognitive strategies.

> In most cases, students with achievement levels below mid-third to fourth grade will have difficulty generalizing the use of formally taught cognitive strategies. (p. 3)

In other words, haphazard use of the cognitive strategies approach may result in adverse effects.

This study suggests that the common assumption of strategic deficiency found among the learning disabled needs to be redefined and verified. Cawley (1985) commented that:

> the present literature in learning disabilities indicates that no matter what the comparison, learning disabled children perform less adequately than nonlearning disabled children. It seems that the research needs to specify the conditions and tasks under which these differences are minimized or disappear. Without such information, it is difficult to design curriculum and instructional programs or to make astute placement decisions. (p. 57)

Suggestion also extends to employ a cognitive- or process-oriented instructional approach based upon teaching learning and thinking strategies. The major rationale for this lies in teaching the learning disabled to know how and when to execute appropriate strategies across various kinds of tasks and situations. It supports Case's (1985) notion that individuals need to have a set of more global cognitive and affective processes which allow them to orchestrate them properly. His idea can be applied to advocate facilitating the learners to develop and generate insight as well as conduct their own cognitive and affective processes applicable across tasks domains and situations.

Research focus on learning disabilities needs to shift from addressing the weaknesses found among these heterogeneous special populations to exploring their strengths. How do the interactions of person, task, and environmental characteristics influence the executive processes of strategies? How do the M.L.D.s modify flexibly some learned or self-generated strategies and a set of well-established tactics, such as the basic mathematics skills, when confronted with novel task demands or situations? To what extent do the affective and cognitive aspects of learning experience influence the quality of the M.L.D.'s, task performance? If the proliferation of fuzzy studies in learning disabilties can be reduced by means of focusing on narrowly-defined research questions and clearly-defined target populations, this becomes the mandate of special educators and practitioners.

Bibliography

Bibliography

Algozzine, B. & Ysseldyke, J. Learning disabilities as a subset of school failure: The oversophistication of a concept. Exceptional Children, 1983, 50, 242-246.

Alley, G. & Deshler, D. Teaching the Learning Disabled Adolescent: Strategies and Methods. Denver: Love Publishing Co., 1979.

Allport, G.W. Pattern and Growth in Personality. New York: Holt, Rinehart and Winston, 1961.

Austin, J.D. Children with a learning disability and mathematics. School Science and Mathematics, 1982, 82, 201-209.

Bartell, N.R. Problems in arithmetic achievement. In Hammill, D.D. & Bartell, N.R. (Eds.) Teaching Children with Learning and Behavior Problems. Boston: Allyn and Bacon, 1975, 99-143.

Bell, A.W. Mathematics in the middle years. In Wilson, B. (Ed.) Mathematics Education: Anglo-Soviet Seminar. London: The British Council, 1981, 57-69.

Bishop, A.J. Research in mathematics education in the UK. In Wilson, B. (Ed.) Anglo-Soviet Seminar: Mathematics Education. London: The British Council, 1981, 88-91.

Blankenship, C.S. A behavioral view of mathematical learning problems. In Cawley, J.F. (Ed.) Cognitive Strategies and Mathematics for the Learning Disabled, Rockville, M.D.: Aspen Systems Corporation, 1985, 40-73.

Bley, N.S. & Thornton, C.A. Teaching Mathematics to the Learning Disabled. Rockville, M.D.: Aspen Systems Corporation, 1981.

Bogolyulov, A.N. A combined analytic and synthetic method of solving arithmetic problems in elementary school. In Kilpatrick, J. & Wirszup, I. (Eds.) Soviet Studies in the Psychology of Learning and Teaching Mathematics, The University of Chicago, 1972, 6, 61-94.

Bo[illegible]kowski, J.G. Signs of intelligence: Strategy generalization and metacognition. In Yussen, S.R. (Ed.) The Growth of Reflection in Children, New York: Academic Press, Inc., 1985, 105-144.

Brandt, R. Studying Behavior in Natural Settings. New York: Holt, Rinehart & Winston, Inc., 1972.

Brown, A.L. Knowing when, where, and how to remember: A problem of metacognition. In Glaser, R. (Ed.) Advances in Instructional Psychology, Vol. 1, Hillsdale, N.J.: Lawrence Erlbaum, 1978.

Brown, A.L. & Palincsar, A. Inducing strategic learning from texts by means of informed, self-control training. Topics in Learning and Learning Disabilities, 1982, 2, 1-18.

Brown, A.L. & Smiley, S.S. The development of strategies for studying texts. Child Development, 1978, 49, 1076-1088.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.M. & Reys, R.E. National assessment: A perspective of students' mastery of basic mathematics skills. In Lindquist, M.M. (Ed.) Selective Issues in Mathematics Education. Berkeley, Calif.: McCutchan Publishing Corporation, 1980, 215-223.

Case, R. Piaget and beyond: Toward a developmentally based theory and technology of instruction. In Glaser, R. (Ed.) Advances in Instructional Psychology, Vol. 1, Hillsdale, New Jersy: Lawrence Erlbaum Associates, 1978.

Case, R. The process of stage transition: A neo-Piagetian view. In Sternberg, R.J. (Ed.) Mechanisms of Cognitive Development, New York: Freeman, 1984, 20-44.

Cawley, J.F. Learning and achievement: Implications for mathematics and learning disability. Focus on Learning Problems in Mathematics, 1985, 7, 49-59.

Cawley, J.F. Cognition and the learning disabled. In Cawley, J.F. (Ed.) Cognitive Strategies and Mathematics for the Learning Disabled, Rockville, M.D.: Aspen Systems Corporation, 1985, 1-32.

Ceci, S.J. & Peters, D. Dyscalculia and the perceptual deficit hypothesis: A correlational study. Focus on Learning Problems in Mathematics, 1980, 2, 11-15.

Chatman, J., Johnsen, E., Tollefson, N. & Tracy, D. Teaching Learning Disabled Students Goal Implementation Skills. Institute for Research in Learning Disabilities. The University of Kansas, 1983.

Chi, M.T.H. Knowledge structures and memory development. In Seigler, R. (Ed.) Children's Thinking: What Develops ? Hillsdale, N.J.: L. Erlbaum Associates, 1978.

Cohn, R. Dyscalculia. Archives of Neurology, 1961, 4, 301-307.

Cohn, R. Developmental dyscalculia: In developmental disorders of mobility and language. Pediatric Clinics of North America, 1968, 15, 651-658.

Covington, M., Crutchfield, R., Davies, L. & Olton, R. The Productive Thinking Program: A Course in Learning to Think. Columbus, Ohio: Merrill Publishing Co., 1972.

Critchley, M. The Dyslexic Child. Springfield, Illinois: Charles C. Thomas, 1970.

Cruickshank, W.M., Deutsch, C.P., Morency, A., Strother, C.R. & Wepman, J.M. Learning disabilities. In Hobbs, N. (Ed.) Issues in the Classification of Children. Vol. 1, San Francisco: Jossey-Bass, 1975.

Cullinan, D., Epstein, M.H. & Silver, L. Modification of impulsive tempo in learning disabled pupils. Journal of Abnormal Child Psychology, 1972, 5, 437-444.

Days, H.C., Kulm, G. & Wheatley, G.H. Problem structure, cognitive level, and problem-solving performance. Journal for Research in Mathematics Education, 1979, 10, 135-146.

Dennis, C., Foster, C. & Maxwell, J. Teaching Self-Control Procedures to Learning Disabled Youths. Institute for Research in Learning Disabilities. The University of Kansas, 1981.

DeRuiter, J.A. & Wansart, W.L. Psychology of Learning Disabilities: Applications and Educational Practice. Rockville, M.D.: Aspen Systems Corporation, 1982, 209.

Deshler, D.D., Alley, G.R., Warner, M.M. & Schumaker, J.B. Instructional Practice That Promote Acquisition and Generalization of Skills by Learning Disabled Adolescents. Institute for Research in Learning Disabilities. The University of Kansas, 1980, 1-18.

Deshler, D.D., Schumaker, J.B. & Lenz, B.K. Academic and cognitive interventions for learning disabled adolescents: Part 1 Journal of Learning Disabilities, 1984, 17, 108-117.

Digate, G., Epstein, M.H., Cullinan, D. & Switzky, H.N. Modification of impulsivity: implications for improved efficiency in learning for exceptional children. Journal of Special Education, 1978, 12, 459-468.

Elkins, J. Learning disability research: Structured reactions to random thoughts about a fussy subject. The Journal of Special Education, 1976, 10, 149-155.

Ericsson, K.H. & Simon, H.A. Verbal reports as data. Psychological Review, 1980, 87, 215-251.

Felton, V.C. An Investigation of the Relationship between Convergent Thinking, Divergent Thinking and Problem Solving Performance in Young Children. Unpublished Doctoral Dissertation, University of California, Berkeley, 1981.

Feuerstein, R. The Dynamic Assessment of Retarded Performers: The Learning Potential Assessment Device, Theory, Instruments, and Techniques. Baltimore: University Park Press, 1979.

Feuerstein, R. & Hoffman, M.B. Intergenerational conflict of rights: Cultural imposition and self-realization. Viewpoints in Teaching and Learning, 1982, 58, 44-63.

Flavell, J.H. Developmental studies of mediated memory. In Reese, H.W. & Lipsitt, L.P. (Eds.) Advances in Child Development and Behavior, Vol. 5, New York: Academic Press, 1970.

Flavell, J.H. Metacognitive aspects of problem solving. In Resnick, L.B. & Glaser, R. (Eds.) The Nature of Intelligence. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1976.

Flavell, J.H. Metacognitive development. In Scandura, J.M. & Brained, C.J. (Eds.) Structural/Process Theories of Complex Human Behavior. The Netherlands: Sijthoft & Noordoff, 1978.

Flavell, J.H. Metacognitive and cognitive monitoring: A new area of cognitive development inquiry. American Psychologist, 1979, 39, 906-911.

Flavell, J.H., Friedrichs, A.G. & Hoyt, J.D. Developmental changes in memorization processes. Cognitive Psychology, 1970, 1, 324-340.

Flavell, J.H. & Wellman, H.M. Metamemory. In Kail, R.V., Jr. & Hagen, O.W. (Eds.) Perspectives on the Development of Memory and Cognition. Hillsdale, N.J.: Lawrence Erlbaum, 1977.

Forrest-Pressley & Waller, T.G. Knowledge and monitoring abilities of poor readers. Topics in Learning and Learning Disabilities, 1984, 3, 73-79.

Gagne, R.M. Some issues in the psychology of mathematics instruction. Journal for Research in Mathematics Education, 1983, 4, 7-18.

Garner, R. Verbal-report data on reading strategies. Journal of Reading Behavior, 1982, 14, 159-167.

Gerber, M.M. Learning disabilities and cognitive strategies: A case for training or constraining problem solving? Journal of Learning Disabilities, 1983, 16, 255-259.

Glass, A.L., Hoyoak, K.J. & Santa, J.L. Cognition. Don Mills, Ontario: Addison-Wesley, 1979.

Gold, A.P. Cumulative Learning Versus Cognitive Development: A Comparison of Two Different Theoretical Bases for Planning Remedial Instruction in Arithmetic. Unpublished Doctoral Dissertation, University of California, Berkeley, 1978.

Goodman, E. A day in the resource room with a minicomputer. Journal of Learning Disabilities, 1983, 16, 117-118.

Goodstein, H.A. Are errors we see the true errors? Error analysis in verbal problem solving. Topics in Learning and Learning Disabilities, 1981, 1, 31-45.

Gottlieb, J. & Strichart, S.S. A perspective on learning disabilities research. In Gottlieb, J. & Strichart, S.S. (Eds.) Developmental Theory and Research in Learning Disabilities. Baltimore: University Park Press, 1981, 310-326.

Grimes, L. Computers are for kids: Designing software programs to avoid problems of learning. Teaching Exceptional Children, 1981, Nov., 49-53.

Hagin, R.A., Beecher, R. & Silver, A.A. Definition of learning disabilities: A clinical approach. In Das, J.P., Mulcahy, R.F. & Wall, A.E. (Eds.) Theory and Research in Learning Disabilities. New York: Plenum Press, 1982, 45-57.

Hallahan, D.P., Hall, R.J., Ianna, S.O., Kneedler, D.R., Lloyd, J.W., Loper, A.B. & Reeve, R.E. Summary of research findings at the University of Virginia Learning Disabilities Research Institute. Exceptional Education Quarterly, 1983, 4, 95-111.

Hallahan, D.P., Kauffman, J.M., Tarver, S.G. & Ball, D.W. Verbal rehearsal and selective attention in children with learning disabilities: a developmental lag. Journal of Experimental Child Psychology, 1976, 22, 375-385.

Hallahan, D.P. & Reeve, R.E. Selective attention and distractibility. In Keogh, B.K. (Ed.) Advances in Special Education. Vol. 1, Greenwich, Conn.: JAI Press, 1980.

Hallahan, D.P. & Sapona, R. Self-monitoring of attention with learning disabled children: Past research and current issues. Journal of Learning Disabilities, 1983, 16, 619.

Hammill, D. & Bartel, N. Teaching Children with Learning and Behavior Problems. Boston: Allyn and Bacon, 1974.

Hammill, D., Leigh, J.E., Mcnutt, G. & Larsen, S.C. A new definition of learning disabilities. Learning Disabilities Quarterly, 1981, 4, 336-342.

Haring, N.G. & Bateman, B Teaching the Learning Disabled Child. Englewood Cliffs, New Jersey: Prentice-Hall, 1977.

Hayes, J.R. The Complete Problem Solver. Philadelphia, Pa.: Franklin Institute Press, 1981, 51.

Henderson, K.B. & Pingry, R.E. Problem solving in mathematics. The learning of mathematics: its theory and practice. Twenty-First Yearbook of the National Council of Teachers of Mathematics. Washington D.C.: The Council, 1953, 228-270.

Homans, G. Sentiments and Activities: Essays in Social Science. New York: Free Press of Glencoe, 1962.

Hudgins, B.B. *Learning and Thinking: A Primer for Teachers*. Itasca, Ill.: F. E. Peacock Publishers, Inc., 1977.

Iwata, B.A., Neef, N.A. & Page, T.J. Teaching pedestrian skills to retarded persons: Generalization from the classroom to the natural environment. *Journal of Applied Behavior Analysis*, 1976, *9*, 433-444.

Jansky, J. The phenomenon of plasticity in relation to manipulating numbers and early learning of arithmetic. In Hellmuth, J. (Ed.) *Learning Disorders*. Vol. 1, Seattle: Straub and Hellmuth, 1965.

Johnson, D.J. & Myklebust, J.R. *Learning Disabilities: Educational Principles*. New York: Grune and Stratton, 1967.

Johnson, S.W. *Arithmetic and Learning Disability — Guidelines for Identification and Remediation*. Boston: Allyn and Bacon, Inc., 1979.

Kagan, J. & Kogan, N. Individual variation in cognitive processes. In Mussen, P.H. (Ed.) *Carmichael's Manual of Child Psychology*, Vol. 1, New York: Wiley, 1970.

Kalmykova, Z.I. Processes of analysis and synthesis in the solution of arithmetic problems. In Kantowski, M.G. (Ed.) *Soviet Studies in the Psychology of Learning and Teaching Mathematics*. The University of Chicago, 1975, *11*, 5-37.

Kantowski, M.G. Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education*, 1977, *8*, 163-180.

Kazdin, A.E. *Research Design in Clinical Psychology*. New York: Harper & Row, Publishers, 1980.

Kazdin, A.E. *Single-case Research Designs: Methods for Clinical and Applied Settings*. New York: Oxford University Press, Inc., 1982.

Keogh, B.K. Perceptual and cognitive style: Implications for special education, In Mann, L. & Sabatino, D. (Eds.) *The First Review of Special Education*, Vol. 1, Philadelphia: JSE Press, 1973.

Keogh, B.K. Another way to drown in the name of science: A response to S. Alan Cohen's proposal solution to research problems in learning disabilities. The Journal of Special Education, 1976, 10, 137-139.

Keogh, B.K. & Margolis, J. Learn to labor and to wait: Attentional problems of children with learning disorders. Journal of Learning Disabilities, 1976, 9, 18-28.

Kirby, J.R. Strategies and processes. In Kirby, J.R. (Ed.) Cognitive Strategies and Educational Performance. Orlando, Flo.: Academic Press, Inc., 1984, 89-109.

Kirby, J.R. Educational roles of cognitive plans and strategies. In Kirby, J.R. (Ed.) Cognitive Strategies and Educational Performance. Orlando, Flo.: Academic Press, Inc., 1984, 89-109.

Kosc, L. Developmental dyscalculia. Journal of Learning Disabilities, 1974, 7, 164-177.

Lawson, M.J. Being executive about metacognition. In Kirby, J.R. (Ed.) Cognitive Strategies and Educational Performance. Orlando, Flo.: Academic Press, Inc., 1984, 89-109.

Lester, F.K. Jr. Problem solving: Is it a problem ? In Lindquist, M.M. (Ed.) Selective Issues in Mathematics Education. Berkeley, Calif.: McCutchan Publishing Corporation, 1980, 29-45.

Levy, W.K. Dyscalculia: Critical analysis and future directions. Focus on Learning Problems in Mathematics, 1979, 1, 1-11.

Liedtke, W. Primary mathematics: Suggestions for diagnosis and remediation. Special Education in Canada, 1979, 54, 13-15.

Lucas, J.F., Branca, N., Goldberg, D., Kantowski, M.G., Kellogg, H. & Smith, J.P. A process-sequence coding system for behavioral analysis of mathematical problem solving. In Goldin, G.A. & McClintock, C.E. (Eds.) Task Variables to Mathematical Problem Solving. Columbus, Ohio: Eric/SMEAC, 1979.

Lupart, J. & Mulcahy, R. Some thoughts on research in learning disabilities and attention. In Kirby, J.R. (Ed.) Cognitive Strategies and Educational Performance. Orlando, Flo.: Academic Press, Inc., 1984, 217-247.

Mayer, R.E. Information processing variables in learning to solve problems. Review of Educational Research, 1975, 45, 525-541.

McEntire, E. Learning disabilities and mathematics. Topics in Learning and Learning Disabilities, 1981, 1, 1-15.

Messinger, M. Computer literate LD students shine at Lakewood Elementary School in Silicon Valley, CA. Journal of Learning Disabilities, 1983, 16, 426-427.

Meichenbaum, D. Self-instructional method. In Kanfer, F.H. & Goldstein, A.P. (Eds.) Helping People Change: A Textbook of Methods. New York: Pergamon Press, Inc., 1975, 357-391.

Meichenbaum, D. Cognitive Behavior Modification: An Integrative Approach. New York: Plenum Press, 1977.

Meichenbaum, D. Metacognitive assessment. In Yussen, S. (Ed.) The Development of Insight in Children. New York: Academic Press. In press.

Mikhal'skii, K.A. The solution of complex arithmetic problems in auxiliary school. In Kilpatrick, J. & Wirszup, I. (Eds.) Soviet Studies in the Psychology of Learning and Teaching Mathematics. The University of Chicago, 1975, 9, 16-91.

Monakhov, V.M. Scientific and methodological research into problems concerning the improvement of mathematical education in the U.S.S.R. In Wilson, B. (Ed.) Mathematics Education: Anglo-Soviet Seminar. London: The British Council, 1981.

Muth, K.D. Solving arithmetic word problems: Role of reading and computational skills. Journal of Educational Psychology, 1984, 7, 205-210.

National Advisory Committee on Handicapped Children. Special Education for Handicapped Children (First Annual Report). Washington, D.C.: U.S. Department of Health, Education and Welfare, 1968.

Newell, A. & Simon, H.A. Human Problem Solving. Englewood Cliffs, N.J.: Prrentice-Hall, Inc., 1972.

Orstein, P.A., Naus, M.J. & Liberty, C. Rehearsal and organizational processes in children's memory. Child Development, 1975, 45, 818-830.

Owings, R.A., Peterson, G.A., Bransford, J.D., Morris, C.D. & Stein, B.S. Spontaneous monitoring and regulation of learning: A comparison of successful and less successful fifth graders. *Journal of Educational Psychology*, 1980, *72*, 250-256.

Paris, S.G. & Myers, M. Comprehension monitoring, memory, and study strategies of good and poor readers. *Journal of Reading Behavior*, 1981, *13*, 5-22.

Paris, S.G., Cross, D., DeBritto, A.M., Jacobs, J., Oka, E, & Saarnio, D. Improving children's metacognition and reading comprehension with classroom instruction. In Research colloquium presented at the annual meeting of the American Educational Research Association, New Orleans, LA., April 24, 1984.

Pieper, E. & Deshler, D.D. Intervention consideration in mathematics for the L.D. adolescent. *Focus on Learning Problems in Mathematics*, 1985, *7*, 35-47.

Polya, G. *How To Solve It*. New York: Doubleday & Company, Inc., 1957.

Polya, G. *Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving*, New York: John Wiley & Sons Inc., 1970.

Proverbs 20:5, New American Standard Bible, New York: The Lockman Foundation, 1971.

Reisman, F.K. & Kauffman, S.H. *Teaching Mathematics to Children with Special Needs*. Columbus, Ohio: Charles E. Merrill Publishing Co., 1980.

Roberts, G.H. The failure strategies of third grade arithmetic pupils. *The Arithmetic Teacher*, 1968, *15*, 442-446.

Schiffman, G., Tobin, D. & Buchanan, B. Micro-computer instruction for the learning disabled. *Journal of Learning Disabilities*, 1982, *15*, 557-559.

Schoenfeld, A.H. Episodes and executive decisions in mathematical problem-solving. In Lesh, R. & Landau, M. (Eds.) *Acquisition of Mathematics Concepts and Processes*. New York: Academic Press, 1983, 345-395.

Schumaker, J.B., Deshler, D.D., Alley, G.R. & Warner, M.M. Toward the development of an intervention model for learning disabled adolescents: The University of Kansas

Institute. Exceptional Education Quarterly, 1983, 4, 45-74.

Senf, G.M. Memory and attention factors in specific learning disabilities. Journal of Learning Disabilities, 1971, 4, 94-106.

Shchedrovitskii, G.P. Toward the analysis of problem-solving processes. In Kilpatrick, J. & Wirszup, I. (Eds.) Soviet Studies in the Psychology of Learning and Teaching Mathematics. The University of Chicago, 1972, 6, 111-118.

Sheinker, A., Sheinker, J.M. & Stevens, L.J. Cognitive strategies for teaching the mildly handicapped. Focus on Learning Problems in Mathematics, 1985, 7, 35-47.

Shepherd, M.J., Fleischner, J. & Gettinger, M.B. Progress Report: General Learning Disabilities Task Force. Research Institute for the Study of Learning Disabilities. Teachers College, Columbia University, 1979.

Simon, D.P. & Simon, H.H. Individual differences in solving physics problems. In Seigler, R. (Ed.) Children Thinking: What Develops ? Hillsdale, N.J.: Erlbaum, 1978.

Solso, R.L. Cognitive Psychology. New York: Harcourt Brace Jovanovich, Inc., 1979.

Steffe. L.P., Richards, J. & Von Glasserfeld, E. Experimental models for the child's acquisition of counting and of addition and subtraction. In Geeslin, W. (Ed.) Explorations in the Modeling of the Learning of Mathematics. Columbus, Ohio: Eric/SMEAC, 1979.

Sternberg, R.J. Mechanisms of Cognitive Development. New York: W.H. Freeman, 1984.

Swanson, H.L. A multidirectional model for assessing learning disabled students' intelligence. Learning Disabled Quarterly, 1982, 5, 316-326.

Swanson, H.L. & Watson, B.L. Educational and Psychological Assessment of Exceptional Children: Theories, Strategies, and Applications. The C.V. Mosby Company, 1982.

Swanson, H.L. & Cooney, J.B. Strategy transformation in learning disabled and nondisabled students. Learning Disability Quarterly, 1985, 8, 221-230.

Torgesen, J.K. The role of nonspecific factors in the task performance of learning disabled children: A theoretical assessment. Journal of Learning Disabilities, 1977, 10, 27-35.

Torgesen, J.K. & Silver, K. Factors related to poor performance of reading disabled children on rote memory tasks. Learning Disabilities Research Quarterly, 1979, 2, 17-23.

Torgesen, J.K., Murphey, H.A. & Ivey, C. The influence of an orienting task in the memory performance of children with reading problems. Journal of Learning Disabilities, 1979, 12, 396-401.

Torgesen, J.K. & Licht, B.G. The learning disabled child as an inactive learner: Retrospect and prospects. In McKinney, J.D. & Feagans, L. (Eds.) Current Topics in Learning Disabilities, Vol. 1, Norwood, New Jersey: Ablex, 1983, 3-31.

Torgesen, J.K. & Young, K.A. Priorities for the use of microcomputers with learning disabled children. Journal of Learning Disabilities, 1983, 16, 234-237.

Torrance, E.P. Torrance Tests of Creative Thinking: Norms-Technical Manual. Princeton, N.J.: Personnel Press, 1966.

Ulibarri, D.M. Cognitive Processing Theory and Culture-Loading: A Neo-Piagetian Approach to Test Bias. Unpublished Doctoral Dissertation, University of California, Berkeley, 1982.

Vygotsky, L.S. Thought and Language. New York: Wiley, 1962.

Vygotsky, L.S. The development of perception and attention. In Cole, M., John Steiner, V., Scribner, S. & Souberman, E. (Eds.) Mind in Society. Cambridge: Harvard University Press, 1978, 31-37.

Vygotsky, L.S. Interaction between learning and development. In Cole, M., John Steiner, V., Scribner, S. & Souberman, E. (Eds.) Mind in Society. Cambridge: Harvard University Press, 1978.

Webb, N.L. Processes, conceptual knowledge, and mathematical problem-solving ability. Journal for Research in Mathematics Education, 1979, 10, 83-93.

Wheatley, C.L. Calculator use and problem-solving performance. Journal for Research in Mathematics Education, 1980, 11, 323-334.

Wiens, J.W. Metacognition and the adolescent passive learner. Journal of Learning Disabilities, 1983, 16, 145-149.

Willems, E.P. An ecological orientation in psychology. Merrill-Palmer Quarterly of Behavior and Development, 1965, 11, 317-343.

Wolpert, E.M. Understanding Research in Education. New York: Kendall/Hunt Publishing Co., 1981.

Wong, B.Y.L. The role of theory in learning disabilities research: Part II. A selective review of current theories of learning and reading disabilities. Journal of Learning Disabilities, 1979, 2, 15-24.

Wong, B.Y.L. Strategic behaviors in selecting retrieval cues in gifted, normal achieving and learning disabled children. Journal of Learning Disabilities, 1982, 15, 33-38.

Appendix A

Instruction

Researcher will give the following instruction before starting the assessment:

I am interested in knowing how the 7th graders will think while solving some mathematics word problems.

You are going to think aloud, which means to talk/say out loudly everything that comes to your mind, while writing down how to solve each mathematics problem.

You might want to use some scratch papers but be sure to write down each item's number.

This is not a test. No grades will be given.

Do you have any questions about the instruction?

Appendix B

Task A word problems

1. Mr. Edwards paid $3260 for a stereo set.
   He also spent $457 for buying a rocking chair.
   How much did he pay in all?

2. Peter picked peaches for 16 days and earned $384 last month. He also earned $95 for picking apricots this month. How much did he earn each day for picking peaches last month?

3. A school library ordered 16 records, some books, and 7 posters. 9 of the books and 16 records cost $270. 7 posters cost $24. How much would it cost for all the records, books, and posters?

4. A building has 826 people living in 130 units.
   An average of 4 people live in each unit.
   There are 16 units on each floor. 147 more people will move in the building this month.
   How many people will be found in the building?

5. There are 735 notebooks in 15 boxes.
   How many notebooks are found in each box?

6. Many rows of seats are found in a music hall.
   685 people filled up some of the seats.
   How many seats are found in each row?

Task B word problems

1. Mrs. Murphy bought 24 bags of potatoes. Each bag has 83 potatoes. How many potatoes are found in all the bags?

2. Tom took out $768 from his saving box. He bought his sister a birthday gift and also spent $359 for buying a bike last week. He bought a pair of skating shoes and paid $27 for 2 records this week. How much did he spend in 2 weeks?

3. Kristy has 15 stamp albums. She has 79 U.S. stamps. Each album holds 129 stamps. Two of the stamp albums have 36 Canadian stamps. How many stamps does she have in all the albums?

4. Jerry paid $748 for his phone bill, housing rent, and grocery bill. He also paid $14 monthly rent for his phone and $110 for painting the door of his house. The phone bill and the grocery bill were $139. How much did he pay for the housing rent?

5. Lucy had $2595 last week. She paid $628 for buying a T.V. How much did she have left?

6. Jeff did 12 push-ups each day last week. decided to do more than 15 push-ups each day this week but he stopped doing it for a few days. How many push-ups did Jeff do this week?

Appendix C

Probing questions

General probing questions

1. What are you trying/going to find out?
2. How are you going to figure it out?
3. What are you thinking?
4. How did you get this answer?

Specific probing questions for problems with surplus information

1. Did you use all the information given?

   If subject says "yes" after being asked the first question, then continue to ask:

   a. What information did you use?
   b. Why would you need it to solve the problem?
   c. How/what did you do with the information you chose?
   d. Why would you use addition/division/subtraction/multiplication to solve the problem?

   If subject says "no" after being asked the first question, then continue to ask:

   e. What information did you not use?
   f. Why didn't you use it?
   g. How/what did you do with the information you chose?

   If subject indicates only 1 surplus element in a problem, researcher will ask:

h. Was it the only information you didn't use?

Researcher will repeat asking questions e, f, and g if subject says "no" or indicates another surplus element found in the same problem, before continuing to ask:

i. Why would you use addition/division/subtraction/multiplication to solve the problem?

If subject expresses verbally: "There is too much information" or "It is so confusing!", Researcher will ask:

j. How are you going to figure it out? Or

k. What is so confusing?

Specific probing questions for problems with missing information

1. Do you have all the information needed to solve the problem?

If subject says "yes" after being asked the first question, then continue to ask:

a. How/what did you do with the information?

b. Why would you use addition/division/subtraction/multiplication to solve the problem?

c. Do you think this problem can be solved?

If subject says "no" after being asked the first question, then continue to ask:

d. What information are you looking for?

e. Can you write down what you are looking for?
   (Need to specify that incomplete sentences
   will be accepted.)

f. Why would you need it to solve the problem?

g. Can you solve the problem if this/these
   piece/s of information would be given?
   This question will also be asked if subject
   indicates only 1 missing element for the
   same question as well as asking how to
   solve it.

Questions e, f, and g will be repeated if subject indicates another missing element found in the same problem.

Appendix D

Cognitive strategies and task performance

Problems	1	2	3	4	5	6	Total
Strategies							
a. Indicates planning							
b. Rereads the problem							
c. Restates the problem in own words							
d. Draws a diagram							
e. Underlines information given							
f. Makes systematic trial-and-error							
g. Makes random trial-and-error							
h. Makes hypothesis							
i. Recognizes surplus information							
j. Indicates surplus information							
k. Recognizes missing information							
l. Indicates missing information							
m. Checks/corrects step/s of solution							
Outcome							
Verbalizations correspond to the written scripts							

Appendix E

Error patterns

Error Patterns	Problems						
	1	2	3	4	5	6	Total
A. Selects parts/all of the numerical data, correctly/incorrectly unrelated to the question.							
B. Applies all numerical data and/plans incorrectly.							
C. Selects data needed/ related to the question but plans incorrectly.							
D. Carries out more than 1-step for the main solution.							
E. Computational error.							

Appendix F

Criteria for evaluating error patterns

Error patterns

A. Selects parts/all (at least 1 surplus element) of the numerical data as found in problems with surplus/missing elements, correctly or incorrectly, unrelated to the question.

B. Applies all of the numerical data which may/may not be related to the question, as found in problems with surplus/missing elements and plans incorrectly.

Plans incorrectly:

1. Misinterprets the problem and/applies data with similar numerical/verbal feature. For examples, $748 and $110 have similar numerical feature. 15 stamp albums and two of the stamp ablums have similar verbal feature.

2. Strives for all possible operations (at least two), inappropriate hypotheses, or numerical/verbal data but produces incorrect solution.

C. Selects data needed as found in problems with surplus/missing elements but plans incorrectly (follows the 2 criteria for incorrect planning as mentioned above).

D. Carries out more than 1-step for the main solution. Systematic/random trial-and-error using multiplication steps to find out the products of division problems will not be evaluated by this criterion.

E. Computational error includes mistakes found in calculation, place value, regrouping and basic mathematical facts.

Appendix G

Verbal protocols

Subject 1 S=Subject R=Researcher

Task A word problems:

Problem #1

S: (Read the problem orally.) So, you had 3260 and 457 and you add them up . . . So, altogether he spent 3717.

R: Hum, Hum.

Problem #2

S: (Read the problem orally.) Peter picked peaches for 16 days . . . (Read the problem orally. Then kept silent after finishing his 1st reading.)

R: What are you thinking?

S: Peter picked peaches . . . (Reread the whole problem. Slowed down his oral reading speed.)

R: What are you thinking?

S: So, you'll divide 384 by 16. Yap, I think that's what I'll do . . . That will be 24. So, approximately he earned 24 dollars each day.

R: Did you use all the information given?

S: Yeah. I didn't need all that information.

R: What information did you not use?

S: I did not need to know how many apricots he picked cause it doesn't ask that. It just asks how much did he earn for picking peaches last month. I don't need any of these information. It's just trying to trick you.

R: Was it the only information that you did not use?

S: Well, Peter picked peaches for 16 days. In these 16 days, he earned 384 dollars in that one month. Then it asks how much did he earn each day for picking peaches. So, you divide those 16 days in 384. You get 24 dollars for each day.

R: So, was it the only information you did not use?

S: Yap, because I don't need to know how many apricots he's picking or anything or how much money that he earned. Cause the question is how much he earned from peaches. So, that means I don't need all that information. It's just trying to throw in some things. Try to trick you.

R: What information did you use?

S: I just used the information that says Peter picked peaches for 16 days and earned 384 dollars last month.

R: Why did you choose the information?

S: O.K. It says Peter picked . . . last month. Then, you read the other information. It says he also earned 95 dollars ... How much did he earn for picking peaches last month? So, you don't need the information of apricots because it is said nothing in the question about apricots. So, you go up to the first line. You read it over and then you see that it says about peaches. Then, it says Peter picked ... 384 dollars last month. That means how much he earned each day for picking peaches last month. So, the information that's in the middle here about the apricots you don't need because there's nothing saying apricot in that question. So, you go up to the peaches integration and divide 16 and 384 and you get your answer. He earned 24 dollars every day on peaches.

R: You said that you didn't use any information about apricots. What would be the information about apricots that you did not use?

S: Well, it's how much it's saying that in 7 more days he spent on picking apricots that month and earned 95 dollars for picking apricots. You don't need to know that.

R: Hum, hum.

S: So, why bother looking at that information, when you know this is all you need.

Problem #3

S: (Read the problem orally.) O.K. 9 of the books and 16 records cost 270 dollars altogether. . . . cost for all the records, books, and posters? O.K. Well, here put your books and records. You've got your cost of two together. You would need to know is to add these two together. So, you've got all your information there for those two. Just add those up . . . (Kept silent.)

R: What are you doing now?

S: I'm adding.

R: What are you adding?

S: I'm adding these together cause it says how much would it cost for all the records.

R: Which these that you are thinking?

S: Well, it says here 9 of the books and just says some books. I presume that 9 of the books is all the books. So, 16 records, 16 records. Your information is right there together for the books and records. Then, you only get your poster left and 7 posters cost 24 dollars. So, you add 24 dollars and 270 dollars. Whatever you get would be how much would it cost for all the records, books, and posters. So, you add these up and you'll get the answer, 294 dollars.

R: Do you have all the information needed to solve the problem?

S: Yeah! You have all the information because it gives you 2 of your products or things into one price. Then, you've only got three products and your other product and the price is right there. So, you have all the products together.

R: What information did you use?

S: I used my title that starts the problem which is a school library ... and 7 posters. So, I know when it says 9 of the books and 16 records cost 270 dollars. Well, I go back to the top lines. It says 16 records. So, I know there's no records left. It says 9 of the books and then it says some books.

It says 7 posters cost 24 dollars. So, I go back up. It says 7 posters, so I know there's no one left. I just add those up and get the price that'll be how much it all cost.

R: How did you do with the information that you chose?

S: Well, it says 270 dollars for 16 records and 9 of the books. So, that takes up to the top that's here. Then, there's only one left. The next line gives you the information about that one left which is 24 dollars. So, you take 24 dollars of the 7 posters. The 270 dollars for the records and books. You add them together. Whatever you get, you'll get your answer for how much it costs altogether.

R: Hum, hum.

Problem #4

S: (Read the problem orally once. Then reread it orally again.) So, that means you have got 826 people living in 130 units. The question is how many people will be found in the building. Well, 826 people live there now. I don't think that it matters what the units right there. Then, it says 147 more people will move in. How many people will be found in the building. Well, all these information that the stuff again that you might not even need because it's saying that so many people living in 130 units. An average of 4 people live in each unit. Well, that really doesn't really matter. But you go to the question that says how many people will be found in this building. There's 826 people living in there first. 147 more come in. So, I think you're just supposed to add them that'll give you enough information ... That will give you 983 people (Error made in placement value).

R: Did you use all the information given?

S: I didn't need it all. Cause I only used that it says a building has 826 people living in 130 units. It doesn't matter if I used the 130 units because it's just trying to trick you by dividing, multiplying or whatever you want to do with it. Giving all these other numbers, throw them in. Trying to trick you. Are just trying to make you use those. Really all you need is to find out how many people living in that building altogether. It says 147 more people are coming. So, you add those together and that'll be how many people will be found in the building.

R: Can you tell me what information did you not use?

S: I don't need to know the information of so many people are living in 130 units, but I do need to know how many people that're living in 130 units. I don't need to know how many people are living in each unit. I don't need to know how many units on each floor. I don't need all that stuff. I just need to know how many people live in that building right at that time. If 147 more people are coming in, then 826 and 147 should be added together. That would be what your answer is.

R: Why would you add these 2 numbers up, 826 and 147?

S: It says a building has 826 people living in right at this time. Then, it tells you there's 147 more people coming. The question is how many people will be found in this building. That means how many people are gonna be starting to live in this place. So, you add 826 and 147, and get your answer.

Problem #5

S: (Read the problem orally.) Well, there are 735 notebooks and there's 15 boxes. The question is how many notebooks are in each box. Gonna to divide 15 boxes into 735 notebooks and it'll give you the answer of how many notebooks are in each box. 15, 5, 75 (1st systematic trial-and-error: 15x5), 15, 4 .. 60 (2nd systematic trial-and-error: 15x4), 15 times 7 (3rd systematic trial-and-error), 8 is 40 (4th systematic trial-and-error), 15 times 9 (5th systematic trial-and-error). So, that means that there's 49 books to each box.

R: Why would you use all these steps in doing multiplication to solve this problem?

S: Because I can't really tell myself what's how many going to that. So, I want to be sure. So, I want to take all these numbers like 4, 5, 7, 8, 9 . . . and see which one that's exactly the number.

Problem #6

S: (Read the problem orally once. Reread the entire problem orally again) Many rows of seats are found in the music hall. O.K., so that tells me that in this music hall, a lot or many rows of seats are found. 685 people . . . Well, I haven't got enough information to tell me. Cause it just says many rows of seats . . . and then, it just says 685 people filled up some of the seats.

R: What information are you looking for?

S: I want to find out how many seats are found in one row, or in such and such amount of rows. Then, I was gonna to find out how many seats are to each row. I haven't got any idea or information. It just says many rows of seats. I don't know how many is many really. Could be 6, 7.

R: Can you write down the information you are looking for?

S: I can't solve the problem because I don't have the information I need. (Verbalizing the sentence while writing it down.)

R: What information do you need?

S: The information I need is; I need to know (Reread the problem quickly before continuing to write.) How many rows have so many seats to get my answer.

R: Is it the only information you need?

S: Well, the only information I need to know. I don't need to know how many people filled up some seats. What I need to know is, maybe how many seats are found in each row. Maybe they can give me there're 7 rows and 42 chairs were altogether. How many seats are found in each row? Well, you maybe divide 7 into 42. Maybe 6 to each row.

R: Why would you divide them?

S: Because it's giving you that there's 42 altogether. Then, if it just gives you that and it says 6 rows. How many seats are in each row. Then, you divide 6 into 42 and get 7.

Verbal protocols

Subject 2   S=Subject   R=Researcher

Task B word problems:

Problem #1

S: (Read the problem orally.) Mrs. Murphy bought 24 bags of potatoes . . . 24 times 83 . . . 1992 potatoes.

R: Why would you do multiplication in here?

S: Cause there's 24 bags of potatoes and each bag has 83 potatoes in it.

R: What number did you get?

S: 1992 potatoes.

R: Hum, hum.

Problem #2

S: (Read the problem orally.) Tom took out 768 dollars . . . O.K., um.

R: What are you thinking?

S: I do subtract, adding to that each of the numbers (She actually did addition before carrying out subtraction as shown in her written script).

R: What are you trying to find out?

S: How much he spent in 2 weeks. O.K., he took out 768 dollars. Then, he spent 359 dollars plus he spent 27 dollars . . . You add the 9 and 7, 16 . . . 386. Subtract from 768 . . . He spent 382 dollars.

R: Do you have all the information needed to solve the problem?

S: Yeah.

R: What information did you use?

S: I used what he spent for the bike, the records, and skating shoes. I subtracted it from the amount he took out. Added those two together.

R: What are these two numbers?

S: He spent 359 dollars for his bike and 27 dollars for 2 records.

R: Why would you add them up?

S: Cause to get the total of what he spent that way you can subtract from the total he took out from his saving box. I took 768 dollars and I subtracted the total which is 386 and I got 382 dollars that he spent in 2 weeks.

R: Why would you do subtraction in here?

S: Because he spent 386 dollars in 2 weeks. The total is 768 which he took out from his saving box.

R: What answer did you get?

S: 382.

Problem #3

S: (Read the problem orally.) . . . How many stamps does she have in all the albums? O.K., Kristy has 15 albums. 79 U.S. and she's got 36 Canadian stamps. You add 79 and 36 together. She's got 115 stamps and divide that into. Divide 15 into 115. 15 times 3 equal (1st systematic trial-and-error) . . . 15 times 7 (2nd systematic trial-and-error) . . . 15 times 8 (3rd systematic trial-and-error) . . . 120, so I go 7 . . . put a decimal. You go 15 into 6 . . . She has 7.6 or estimate of 8 stamps in each album.

R: Did you use all the information given?

S: Yap.

R: What information did you use?

S: I used that she had 79 stamps from U.S. and I added that with 36 Canadian stamps, and I got 115 stamps altogether. Then divided 15 albums into 115 stamps and I got 7.6 or an estimate of 8 in each album.

R: Why would you do division in here?

S: Cause they want to know the question how many stamps she has in all the albums.

R: Why would you add 79 and 36 first before you did division?

S: Cause she's got 79 U.S. stamps and 36 Canadian stamps. I want to find out how many stamps she has altogether.

R: What did you do with the answer 115 in here after adding these two numbers up?

S: I divided 15. Cause she's got 15 albums, and I divided into 115. I ended up getting 7.6 or an estimate of 8 stamps in each album.

R: Why would you have a remainder in here?

S: Well, 10 subtracted from 115 and 105. Got a 100 or 10 left. You have to put a decimal and go down to 0. 6 times 15 is 90 and then has 10 left over. It's just repeating as 6 on top. The answer is just repeating the number.

R: What do you mean by repeating the number?

S: The 6 will keep going on. Probably will take up the whole paper with 10 over again.

R: So, what would be the answer for this question?

S: 7.6 stamps or an estimate of 8 stamps.

R: Hum, hum.

Problem #4

S: (Read the problem orally.) . . . The phone bill and the grocery bill were 139 dollars. How much did he pay for the housing rent? O.K., he's got 748 dollars total. (Kept silent.)

R: What are you trying to find out?

S: He paid 748 dollars total. I'm trying to find out how much he paid for the housing rent. I take 14, 110, and 139 dollars. Then add together . . . 268 dollars (She actually wrote down 263) and subtract from 748 dollars . . . He paid 485 dollars for housing rent.

R: Did you use all the information given?

S: Yap.

R: What information did you use?
S: I used that he had 768. He paid total 748 dollars for his phone bill, housing rent, and grocery bill. It told how much he paid for the monthly rent of his phone and for 110 dollars for painting the door. For the phone bill and the grocery bill added up to 139. I added 110, 14, 139 up and I got 263. Then I subtracted that from 748 which was the total. I got 485.

Problem #5

S: (Read the problem orally.) . . . O.K., 2595 subtract 628 . . . O.K., Lucy had 1967 dollars left.

R: Why would you use subtraction to solve this problem?

S: Cause it asked how much she had left after buying a T.V. She had 1967 dollars left.

R: Hum, hum.

Problem #6

S: (Read the problem orally.) . . . O.K. 12 push-ups each day every week (The information given was "last week"). A few means 3. So 15 times 3 . . . is 45. He had to do 45 push-ups for this 3 days that he missed.

R: Do you have all the information needed to solve this problem?

S: Yeah. I have all the information. First sentence said Jeff did 12 push-ups each day last week. I didn't exactly need.

R: Why didn't you use it?

S: Cause it asked in the sentence how many he had to do this week. He stopped for a few days. He had to do 45 extra push-ups. So, I tried 15 times 3. Cause a few means 3.

R: Why would you think a few means 3?

S: Cause few. Like couple is 2 and few means 3. Sum is 4 and 5. So, I times 3 and I got 45 push-ups that he had to do extra.

R: Do you think this problem can be solved?

S: Yap!

R: What's the answer for that?

S: 45 push-ups.

Verbal protocols

Subject 3 S=Subject R=Researcher

Task A word problems:

Problem #1

S: (Read the problem orally.) Well, 3260 dollars plus 457 dollars equals . . . 3717.

R: Why would you do addition here?

S: To add the 3260 to the 457. To see how much he paid in all. So, Mr. Edwards paid 3717 in all.

Problem #2

S: (Read the problem orally.) Um . . . (Remained silent.)

R: What are you thinking?

S: I'm thinking . . . I'm not sure, ha! But . . . (Remained silent.)

R: What are you trying to find out?

S: I'm trying to figure out how much money he made for picking peaches for 16 days and earned 384 dollars last month. That's the answer to the question. Right?

R: What do you think?

S: Well, ha! He earned 95 dollars for picking apricots ... (Reread the problem.) earn each day for picking peaches last month (the words "peaches" and "last month" were sounded out at a higher pitch and longer duration.) Would be 384 dollars.

R: Did you use all the information given?

S: No.

R: What information did you not use?

S: The apricots.

R: Why didn't you use it?

S: Cause the question never asked you to use it.

R: Why would you write down 384?

S: That tells how much money he made for picking peaches last month.

R: You said that you didn't use the information about apricots.

S: Right.

R: What information about apricots?

S: The information that he earned 95 dollars for apricots and decided to spend 7 more days this month. So, that shouldn't be in there at all.

R: When you said that it shouldn't be in there, what were you trying to find out?

S: Well, I was trying to find out how much money he earned for picking peaches last month.

Problem #3

S: (Read the problem orally. Kept silent for less than 4 seconds.) Well, actually I couldn't find out the answer to that question. It didn't say how many books it had.

R: Why couldn't you?

S: Cause it says some books and then it says 9 of the books and 16 records. So, I've got to find out how many books it had and how much the books cost.

R: Can you write down the information that you are looking for? I will accept incomplete sentences.

S: O.K., what I am looking for. How many books . . . How much . . .

R: What did you write down?

S: How many books and how much they cost.

R: You mean how much do all the books cost?

S: How much does one book cost or all of them.

R: What does it mean by, here, "how much they cost"? (Indicated in his written script.)

S: How much the books cost. O.K., the school library ordered 16 records . . . cost 270 dollars. But, I don't know how many books there were, O.K. Then, it says how much would it cost for all the records, books, and posters. So, you've to add this, this, and this. But, I don't know this.

R: What are these?

S: How many books. O.K., I've got to add the posters to the records, to the books. But I don't know how many books there are.

R: Why would you need to have these 2 pieces of information to solve the problem?

S: Otherwise, it's not logical. It doesn't make any sense.

R: Why wouldn't it make any sense?

S: Cause it wouldn't work. Cause you need more information. You need to know how many books and how much would they cost.

R: You mean how much would each book cost or how much would all of them cost?

S: How much would each book cost.

R: Why would you need to find out how much would each book cost?

S: So, I add it together, and add it to the cost of the records, and add it to the cost of posters, or the some books.

R: Why would you need to add all these numbers up?

S: To find out how much it would cost for records, books, and posters.

R: Can you solve the problem if these 2 pieces of information are given?

S: Yes.

R: How?

S: How? Say, there's 5 books and 13 dollars each. Then, I would add ... multiply 13 times 5 and plus that . . . and that doesn't include the 9.

R: Why would you multiply 13 and 5?

S: Cause there's 5 books and each one cost 13 dollars, or just like be adding. Just like be adding 13 five times, but I multiply it. O.K., I add that to the 9 books and 16 records, and add it to the 7 posters.

Problem #4

S: (Read the problem orally.) O.K. Then, I just add 826 people to the new 147 people . . . So, there'll be 973 people living in the building.

R: Did you use all the information given?

S: No.

R: What information did you not use?

S: An average of 4 people live in each unit and 16 units on each floor.

R: Why didn't you use them?

S: Cause the question says how many people will be found in the building, not how many units or how many family, but how many people.

R: Was it the only information that you did not use?

S: Um . . . Yes.

R: Why would you do addition for these 2 numbers?

S: Cause 826 people and 147 people. The way to put them together that would make sense. Wouldn't minus them or divide them. You plus them.

R: Why would you think that you needed to do addition?

S: Why not?

R: What would this answer tell you?

S: It'll tell me how many people are living in the building.

R: How did you find it out?

S: By adding. By adding 826 people to 147 people.

R: Why would you add them up?

S: To get the total of 973 people.

Problem #5

S: (Read the problem orally.) I take 15 divided by 735 . . . (735-15 were indicated in his written script.) I'll use scrap. Times 15 times 8 ... 120 (1st systematic trial-and-error), nine! (2nd systematic trial-and-error) . . . There'll 49 notebooks in each box.

R: How did you find out the answer?

S: By division.

R: Why would you need to do division to solve the problem?

S: To divide. Well, otherwise you'll just sorting out. Like 15 sections and putting 1, 2, 3, 4 . . . 15 and keep going over that until you reach 735. Division is easier.

R: What would this answer tell you?

S: This answer 49. It tells me how many notebooks are in each box.

Problem #6

S: (Read the problem orally.) Many rows of seats . . . (Whispered while rereading the problem.)

R: What are you thinking?

S: I couldn't tell you the answer.

R: Why can't you?

S: Why can't I? Because there are many rows of seats and you need to know how many seats there are. Um . . . which seats that people filled. Um . . . I need more information.

R: Can you write down the information that you are looking for?
S: Where're seats filled.

R: Can you explain what does it mean by here, "where're seats filled"?

S: Like, there's 30 rows, you have to know how many in each row, 15, 30, 100. O.K., and how many seats and also the size of building, and size of seats.

R: Why would you need to know where're seats filled?

S: Um. I'm thinking that I knew it a minute ago, but I don't know now. Um, I guess you wouldn't need it. (Erased the sentence.)

R: Why would you change your mind?

S: I thought of something out of wasn't something. I thought of something relevant but wouldn't relevant.

R: What were you thinking?

S: Like, when I got the size of the seats, I know how many there were.

R: Why would you erase it?

S: Cause you don't need it. It's just extra information . . . Just give me more information.

R: Why would you need to know how many seats in order to solve the problem?

S: Because I would know how many rows are and the size of the building. Oh! I also have to . . . Space and rows or between rows.

R: You were talking about you needed to know the information of how many seats.

S: Right.

R: So, you were talking about the seats?

S: And how many seats are, so that I'll know how many seats'll fit into the one building. O.K., I'll have to know how many seats'll fit into the size of the building. O.K., I have to know the size of the seats. To see how many seats will fit into the building. I have to see the space between the rows or the seats.

R: Can you solve the problem if all these information would be given?

S: Yes.

R: How?

S: First of all, see the size of the building (Began to draw a diagram.) See the building was like this square, 500 meters. Then, stage came like this. Oh! Yeh! May be they don't have stage. Was it a stage in there?

R: Do you think if there's a stage?

S: Maybe there's a stage. Yeah, size of stage (Wrote it down) O.K.

R: Why would you need to know the size of the stage?

S: See how much room that the size of the stage takes up that 500 meters. Could be 5000 meters. Anyway, then you have probably a row here, a row here . . .

R: What are you drawing?

S: 3 rows . . . Each of these here, rectangle, is a row. You got more rows like that. You have to find out how much space between . . . here and here.

R: Why would you need to know space between rows?

S: Because, say the space is 500 meters. Could be no row. Like, could be any number. Could be just a foot and fit more chairs in. The more space between them, the less chairs you can fit in.

R: Do you think this problem can be solved if all these information would be given?

S: Yeah, I do.

Verbal protocols

Subject 4 S=Subject R=Researcher

Task B word problems:

Problem #1

S: (Read the problem orally.) O.K. There's 24 bags . . . 83 potatoes in each (wrote down the two numbers). O.K. There's 24 bags with 83 potatoes in each. Then, 83 times 24 ... There's one thousand nine-hundred and ninety-two potatoes found in all (started to write while verbalizing it).

R: How did you find it out?

S: O.K. I got the 2 main factors. I times 83 times 24 and I got the answer.

R: Why would you need to do multiplication to solve the problem?

S: Because it's a large number in each.

R: What do you mean by a large number in each?

S: Well, 83 is a large number, so as 24. So you need to times it. Because, if you plus it, it wouldn't come out. If you minus it, it wouldn't come out. If you divide it, it wouldn't come out.

R: Why would you think so?

S: Because they are different factors and they need to even out.

R: What would this answer tell you?

S: That there's one thousand nine-hundred and ninety-eight potatoes in 24 bags.

Problem #2

S: (Read the problem orally. Started to write down 3 numerical data given: 768, 359, 27, but kept silent over 5 seconds.) O.K.

R: What are you thinking?

S: O.K. Nine plus 7 is 16. 16 plus 8 is 24. 2 remainder. 8 plus 3 is 11. So, the answer is 11. Ah, one thousand one hundred and fifty-four. (wrote down while verbalizing) He spent . . . 1154 dollars in 2 weeks.

R: Do you have all the information needed to solve the problem?

S: Yes.

R: What information did you use?

S: I used how much he took out ($768 was given in the problem). Oh! I did not, (actually she did in her written script) and how much he spent on his sister and how much he spent on his bike and how much he paid for 2 records.

R: What did you do with the information you chose?

S: I added it altogether which I shouldn't. O.K. This. You first add these 2 together (359+27) and then you divide them by that (768) .

R: Why would you think that you have to do addition first and then division?

S: Well, I thought that was how much he spent on his sister and then you divide them by that.

R: How would you figure it out?

S: You take 768 dollars. Oops. (crossed out 768 in her written script) 359 dollars and 27 dollars and add them together. 9 plus 7 is 16 . . . So, it's 386 divided by, I mean divided into 768. 386 times

2 equals . . . 12, (1st systematic trial-and-error) So, 777 (She acctually wrote down 772) So, could only go 1 . . . 9 times 6 is 54 (2nd trial-and-error). . . 8 times 6 is 48 (3rd trial-and-error). It's repeated. So, 1.989.

R: What did you come up with?

S: He spent a hundred and ninety-eight dollars and nine cents. No. And ninety cents.

R: First, you had all these 3 numbers adding up and you changed your plan to do addition.

S: Yeah.

R: Why would you need to do addition?

S: First, you added them to find out what the total was . . . The total of these two. The two here that he spent. I added the one that he took out from his saving box and I wasn't supposed to do that. You are supposed to divide by them how much he took out from his saving box.

R: Which number you said that you shouldn't use?

S: 768.

R: And then later on, what information did you use after changing your plan?

S: After I changed my plan? I used 359 and 27.

R: Why would you need them?

S: To find out how much he spent in all. And then I had to divide the number that, that . . . um, he took out his bank.

R: What information did you choose next for doing the division?

S: Yeah. I choose 386 and 768.

R: What did you do with the information that you chose?

S: I divided it and it came out with 198.90.

R: Why would you need to do division to solve the problem?

S: To find how much he spent out of 768.

R: What did you come up with?

S: 198.90

R: What would this answer tell you?

S: That he spent a hundred and ninety-eight and ninety cents in 2 weeks.

Problem #3

S: (Read the problem orally.) 15 . . . (Kept silent while writing down the number.)

R: What are you thinking?

S: I'm thinking that O.K., she has 15 stamp albums and she has 79 U.S. stamps. Each album hold 129 stamps. So, O.K., it would be 17 of them hold 129. 129 times 17 equals... In all the albums, she has 2265.

R: Did you use all the information given?

S: Yes. No, I didn't. I didn't use she has 79 U.S. stamps and 36 Canadian stamps.

R: Why didn't you use them?

S: Because it wasn't needed.

R: Why would you think that it wasn't needed?

S: Because there are 15 stamp albums. O.K., 17 of them have 129 stamps in it, and 2 of them have 36. The question says how many stamps does she have in all the albums and that's the answer.

R: What did you do with the information you chose?

S: O.K., I did 15 times. 13 times 129 and 36 times 2 and I added those two together.

R: Can you tell me what information that you chose?

S: I chose 15 stamp albums, 129 stamps, 2 stamp albums with 36 stamps.

R: Why would you need all these information to solve the problem?

S: Because the stamp albums. You have to find out how many there are in the 13 and how many are in those two and then you add them together to find the answer.

R: What did you do with the information you chose?

S: I, O.K. I took 13 and 129, and times them. Then I times 36 times 2.

R: Can you tell me the first step that you did with the information you chose?

S: 129 times 17 equals 1903.

R: And then what did you do?

S: And then I took that and I times, times 129 that I wanted and I got 1290, and then I added those together. I got 2193 and then I added 72 equals 2265.

R: How did you get the number 72?

S: 72? Oh! I times 36 times 2 and I got 72. Then, I added those two together and I got 2265.

R: Why would you need to use addition to solve the problem?

S: To find the answer.

R: What were you trying to find out for the answer?

S: How many stamps she had in all the albums.

Problem #4

S: (Read the problem orally.) O.K. You divide 139 by 748 (wrote down 748-139) and times 4 (wrote down 139x4: 1st systematic trial-and-error) . . . 139 times 6 (2nd systematic trial-and-error) ... O.K. 139 times 5 (3rd systematic trial-and-error) . . . O.K. 139 times 3 (4th systematic trial-and-error) . . . So, 53 dollars and 19 cents. No, ha! O.K. 39. . .

R: What did you find out?

S: I found out that you can minus it instead of dividing it.

R: Why would you think so?

S: Because if you paid 748 and half of it was for his grocery bill, then you can just minus it.

R: What information did you use at first?

S: I used 139 and 748.

R: Did you use all the information given?

S: No.

R: What information did you not use?

S: I used 110 for painting his door and I . . . Oh! I have to add. I have to add 139.

R: What information did you use at first?

S: 139 and 748.

R: What did you do with this information?

S: I divided it.

R: Why would you need to do division?

S: You don't. You're not supposed to . . . I did it wrong.

R: Why would you think so?

S: Well, I thought so because of the way that it's written out that it mixed me up. Now, you have to add the monthly rent for his phone and for his grocery bill together and then you minus it from 748. So, O.K., 139 plus 14 equal . . . 153 . . . So, he spent 595 for his housing rent.

R: Did you use all the information given?

S: No.

R: What information did you not use?

S: The 110 dollars for painting of his door of his house.

R: Why didn't you use it?

S: Because it wasn't needed.

R: Why would you think that it wasn't needed?

S: Because he paid 748 for his phone bill, housing rent, and grocery bill. The question asked how much did he pay for just his housing rent. It tells you how much he paid for his grocery bill and his phone bill but not his housing rent.

R: What information did you use?

S: I used the 748 for how much he paid for his phone bill, housing rent, and grocery bill. The 139 for his grocery bill and phone bill.

R: What did you do with the information you chose?

S: I added 139 and 14 together, and I got 153.

R: What would this answer tell you? I mean this number 153.

S: How much he paid for his phone bill and grocery bill. Then, I subtracted that from 748 and I got the number that he paid for his housing rent.

R: Why would you need to do subtraction to solve the problem?

S: To find the answer.

R: What answer?

S: 595.

R: What's that answer tell you?

S: That he spent 595 for his housing rent.

Problem #5

S: (Read the problem orally) O.K., you take 2595 and you subtract it. You subtract 628 from it . . . She has 1967 left.

R: How did you find out this answer?

S: I subtracted 628 from 2595.

R: Why would you need to do subtraction to solve the problem?

S: Because it tells you that she bought a T.V. She had 2595 dollars and the T.V. was 628. So, you subtract that 628 from the 2595 and you should have how much she has left.

Problem #6

S: (Read the problem orally.) O.K. 15 times 5 . . . He did 75 push ups.

R: Do you have all the information needed to solve the problem?

S: Yeah.

R: What information did you use?

S: How many he did, he did, the day of this week and how long he stopped doing them for.

R: What number did you choose?

S: 15 and 5.

R: How did you get this number, 5?

S: O.K. There're 7 days in a week and he stopped doing it for 2 days. So, that would be 5 days left and you times 15 times 5 and that's how many he did this week.

R: Why would you need to do multiplication to solve the problem?

S: Because you can't add them. If you add them, you'll get 20 and 20 is not correct answer.

R: So, you think this problem can be solved?

S: Yeah.

Name: Subject 1

Date: 4/17/85

School: X

1. Mr. Edwards paid $3260 for a stereo set. He also spent $457 for buying a rocking chair. How much did he pay in all ? $3717

2. Peter picked peaches for 16 days and earned $384 last month. He also earned $95 for picking apricots. He decided to spend 7 more days for picking apricots this month. How much did he earn each day for picking peaches last month ? $ 24.

3. A school library ordered 16 records, some books, and 7 posters. 9 of the books and 16 records cost $270. 7 posters cost $24. How much would it cost for all the records, books, and posters ? $294

4. A building has 826 people living in 130 units. An average of 4 people live in each unit. There are 16 units on each floor. 147 more people will move in the building this month. How many people will be found in the building ?

983

5. There are 735 notebooks in 15 boxes. How many notebooks are found in each box ?

49 books to a box

6. Many rows of seats are found in a music hall. 685 people filled up some of the seats. How many seats are found in each row ?

I can't solve the problem because I don't have the information I need.

The information I need is: I need to know How many rows have so many seats to get my answer.

1) 3260
-457
#3717

(2) 24 #
16) 384
32
64

(3) 270
+29
#299

(4) 826
147
983

(5) 49
15) 735
60
135
135
6
90

2
15 15
3 4
75 60

3
9
4
15 15 15
7 8 9
105 120 135

Name: Subject 2 Date: 4/17/85

School: X

1. Mrs. Murphy bought 24 bags of potatoes. Each bag has 83 potatoes. How many potatoes are found in all the bags ?

1992 potatoes

2. Tom took out $768 from his saving box. He bought his sister a birthday gift and also spent $359 for buying a bike last week. He bought a pair of skating shoes and paid $27 for 2 records this week. How much did he spend in 2 weeks ?

He spent $382

3. Kristy has 15 stamp albums. She has 79 U.S. stamps. Each album holds 129 stamps. Two of the stamp albums have 36 Canadian stamps. How many stamps does she have in all the albums ?

7.6 or 8 in each

4. Jerry paid $748 for his phone bill, housing rent, and grocery bill. He also paid $14 monthly rent for his phone and $110 for painting the door of his house. The phone bill and the grocery bill were $139. How much did he pay for the housing rent ?

$485 for housing rent

5. Lucy had $2595 last week. She paid $628 for buying a T.V. How much did she have left ?

$1967 left

6. Jeff did 12 push-ups each day last week. He decided to do more than 15 push-ups each day this week but he stopped doing it for a few days. How many push-ups did Jeff do this week ?

45 push-ups

⑥ 12

15
× 3
45

① 24
× 83
72
1920
1992

③ 15

79
+ 36
115

15 × 3 = 45

15 × 7 = 105

15 × 6 = 90

15 × 8 = 120

15) 115.0 = 07.6
105
100
90
10

② $768

~~7~~68
− 3~~8~~6
$382

$359
+ $27
$386

④ $~~7~~48
− 263
$485

14
1~~0~~0
139
$263

⑤ $~~2~~5~~9~~5
− 628
$1967

Name: Subject 3 Date: 4/23/85.

School: Y

1. Mr. Edwards paid $3260 for a stereo set. He also spent $457 for buying a rocking chair. How much did he pay in all ?

$$\begin{array}{r} \$3260 \\ +\ 457 \\ \hline \$3717 \end{array}$$

2. Peter picked peaches for 16 days and earned $384 last month. He also earned $95 for picking apricots. He decided to spend 7 more days for picking apricots this month. How much did he earn each day for picking peaches last month ?

$384

5 $13

3. A school library ordered 16 records, some books, and 7 posters. 9 of the books and 16 records cost $270. 7 posters cost $24. How much would it cost for all the records, books, and posters ?

how many Books

how much they cost

4. A building has 826 people living in 130 units. An average of 4 people live in each unit. There are 16 units on each floor. 147 more people will move in the building this month. How many people will be found in the building ?

```
  826
  147
 ----
  973
```

5. There are 735 notebooks in 15 boxes. How many notebooks are found in each box ?

```
      49
15 ⟌ 735
     60*
    ----
     135
     135
    ----
     0 0 0
```

6. Many rows of seats are found in a music hall. 685 people filled up some of the seats. How many seats are found in each row ?

Size of stage

how many seats

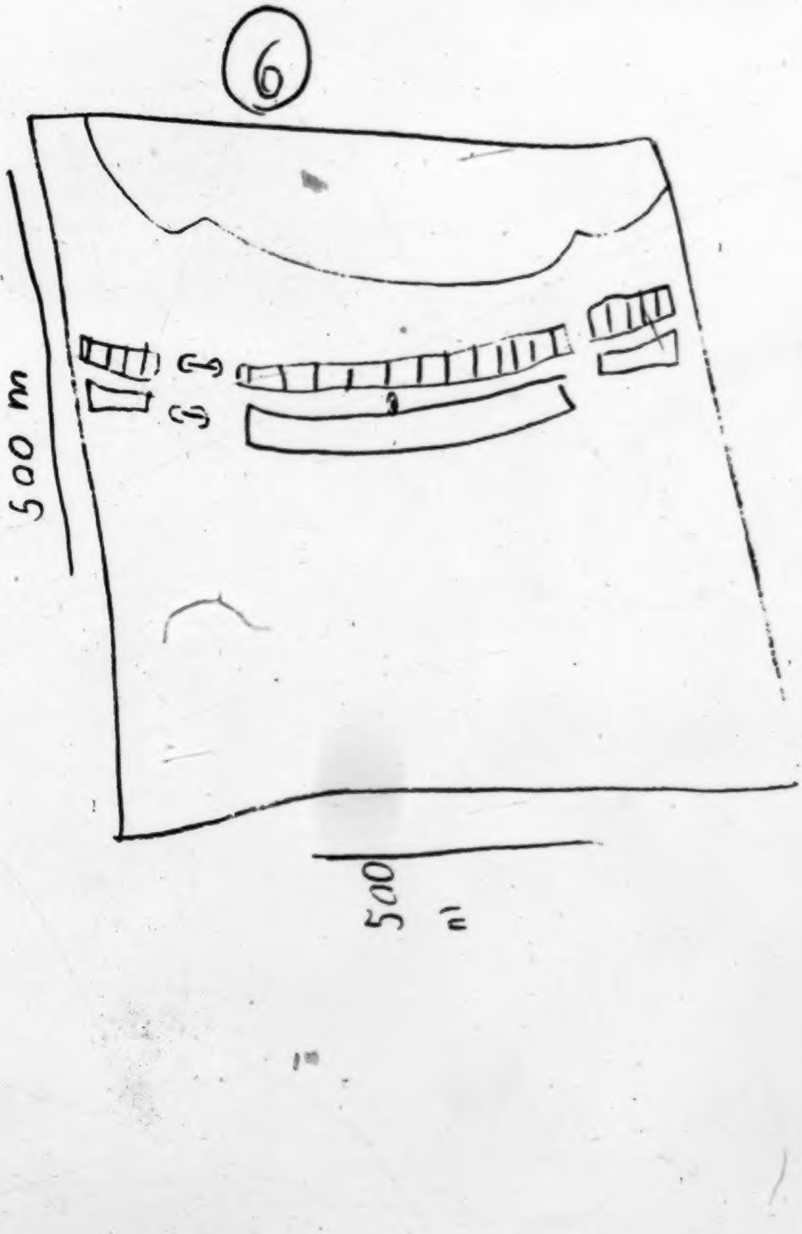
Size of Bulding

Size of seats

Space Between rows

⑤

15
× 8
120



13

Name: Subject 4 Date: 4/23/85

School: Y

1. Mrs. Murphy bought 24 bags of potatoes. Each bag has 83 potatoes. How many potatoes are found in all the bags ?

1992 potatoes found in.

2. Tom took out $768 from his saving box. He bought his sister a birthday gift and also spent $359 for buying a bike last week. He bought a pair of skating shoes and paid $27 for 2 records this week. How much did he spend in 2 weeks ?

he spent $1154.00 in 2 weeks.

1.98.90

3. Kristy has 15 stamp albums. She has 79 U.S. stamps. Each album holds 129 stamps. Two of the stamp albums have 36 Canadian stamps. How many stamps does she have in all the albums ?

2265

4. Jerry paid $748 for his phone bill, housing rent, and grocery bill. He also paid $14 monthly rent for his phone and $110 for painting the door of his house. The phone bill and the grocery bill were $139. How much did he pay for the housing rent ?

595 forhousingrent

5. Lucy had $2595 last week. She paid $628 for buying a T.V. How much did she have left ?

1967

6. Jeff did 12 push-ups each day last week. He decided to do more than 15 push-ups each day this week but he stopped doing it for a few days. How many push-ups did Jeff do this week ?

hedid 75pushups

① 24 bags
83 potatoes in each

748
139

① 83
24
332
1660
1992

15

36
2
72

17

768

② 359
× 27
1154

129
③ × 17
903
1290
2193
72
2265
1989
1386 768.00

② 386
2
772
75
386
9
3474

~~768~~

359
② 27
386

② 386
3820
3474
6 3 3
3 0 88
3820
3474
346

139
14

5.3
139 | 748.00
695
536
417
119

386 3 25
139 139
4 6
556 834

139
5
695

139
3
417

386
7
3088

1

139

14

153

④

6

~~7~~'48

153

5'95

1 8

~~2~~'5~~9~~'5

628

1967 ⑤

2

15

×5

---

72 ⑥